Entrance Examination for M. A. Economics, 2015

Series 01

Time. 3 hours **Maximum marks.** 100

General Instructions. Please read the following instructions carefully:

- Check that you have a bubble-sheet accompanying this booklet. Do **not** break the seal on this booklet until instructed to do so by the invigilator.
- Immediately on receipt of this booklet, fill in your Name, Signature, Roll number and Answer sheet number (see the top left corner of the bubble sheet) in the space provided below.
- This examination will be checked by a machine. **Therefore, it is very important that you follow the instructions on the bubble-sheet.**
- Fill in the required information in Boxes on the bubble-sheet. **Do not write anything in Box 3 - the invigilator will sign in it.**
- Make sure you do **not** have **mobile, papers, books, etc.**, on your person. You can use non-programmable, non-alpha-numeric memory simple calculator. **Anyone engaging in illegal practices will be immediately evicted and that person's candidature will be canceled.**
- You are **not allowed to leave the examination hall** during the first 30 minutes and the last 15 minutes of the examination time.
- When you finish the examination, hand in this **booklet and the bubble-sheet** to the invigilator.

Name ________________ **Signature** ________________

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Before you start

- Check that this booklet has pages 1 through 21. Also check that the top of each page is marked with *EEE 2015 01*. Report any inconsistency to the invigilator.
- You may use the blank pages at the end of this booklet, marked **Rough work**, to do your calculations and drawings. No other paper will be provided for this purpose. Your "Rough work" will be neither read nor checked.

You may begin now. Best Wishes!

Part I

- This part of the examination consists of 20 multiple-choice questions. Each question is followed by four possible answers, at least one of which is correct. If more than one choice is correct, choose **only the 'best one'**. The 'best answer' is the one that implies (or includes) the other correct answer(s). Indicate your chosen best answer on the **bubble-sheet** by shading the appropriate bubble.
- For each question, you will get: 1 mark if you choose only the best answer; 0 mark if you choose none of the answers. **However, if you choose something other than the best answer or multiple answers, you will get $-1/3$ mark for that question.**

Question 1. There are two individuals, 1 and 2. Suppose, they are offered a lottery that gives Rs 160 or Rs 80 each with probability equal to 1/2. The alternative to the lottery is a fixed amount of money given to the individual. Assume that individuals are expected utility maximizers. Suppose, individual 1 will prefer to get Rs 110 with certainty over the lottery. However, Individual 2 is happy receiving a sure sum of Rs 90 rather than facing the lottery. Which of the following statements is correct?

(a) both individuals are risk averse

(b) 2 is risk averse but 1 loves risk
(c) 1 is risk averse but 2 loves risk
(d) none of the above
Answer: (a)

Question 2. Consider an exchange economy with agents 1 and 2 and goods x and y. The agents' preferences over x and y are given. If it rains, 1's endowment is (10, 0) and 2's endowment is (0, 10). If it shines, 1's endowment is (0, 10) and 2's endowment is (10, 0).

(a) the set of Pareto efficient allocations is independent of whether it rains or shines
(b) the set of Pareto efficient allocations will depend on the weather
(c) the set of Pareto efficient allocations may depend on the weather
(d) whether the set of Pareto efficient allocations varies with the weather depends on the preferences of the agents
Answer: (a)

Question 3. Deadweight loss is a measure of
(a) change in consumer welfare
(b) change in producer welfare
(c) change in social welfare
(d) change in social inequality
Answer: (c)

Question 4. To regulate a natural monopolist with cost function $C(q) = a + bq$, the government has to subsidize the monopolist under
(a) average cost pricing
(b) marginal cost pricing
(c) non-linear pricing
(d) all of the above
Answer: (b)

Question 5. Suppose an economic agent lexicographically prefers x to y, then her indifference curves are
(a) straight lines parallel to the x axis
(b) straight lines parallel to the y axis

(c) convex sets
(d) L shaped curves
Answer: (c)

The next two questions are based on the function $f : \Re^2 \to \Re$ given by

$$f(x,y) = \begin{cases} 0, & (x,y) = (0,0) \\ xy/(x^2+y^2), & (x,y) \neq (0,0) \end{cases}$$

for $(x, y) \in \Re^2$.

Question 6. Which of the following statements is correct?
(a) f is continuous and has partial derivatives at all points
(b) f is discontinuous but has partial derivatives at all points
(c) f is continuous but does not have partial derivatives at all points
(d) f is discontinuous and does not have partial derivatives at all points

Question 7. Which of the following statements is correct?
(a) f is continuous and differentiable
(b) f is discontinuous and differentiable
(c) f is continuous but not differentiable
(d) f is discontinuous and non-differentiable

Question 8. Consider the following system of equations:

$$\begin{aligned} x + 2y + 2z - s + 2t &= 0 \\ x + 2y + 3z + s + t &= 0 \\ 3x + 6y + 8z + s + 4t &= 0 \end{aligned}$$

The dimension of the solution space of this system of equations is
(a) 1
(b) 2
(c) 3
(d) 4

Question 9. The vectors $v_0, v_1, \ldots, v_n$ in $\Re^m$ are said to be affinely independent if with scalars $c_0, c_1, \ldots, c_n$, $\sum_{i=0}^{n} c_i v_i = 0$ and $\sum_{i=0}^{n} c_i = 0$ implies $c_i = 0$

for $i = 0, 1, \ldots, n$. For such an affinely independent set of vectors, which of the following is an implication:

I. $v_0, v_1, \ldots, v_n$ are linearly independent.

II. $(v_1 - v_0), (v_2 - v_0), \ldots, (v_n - v_0)$ are linearly independent.

III. $n \leq m$.

(a) Only I and II are true

(b) Only I and III are true

(c) Only II is true

(d) Only II and III are true

Question 10. $\Re^2 \to \Re^2$ be a linear mapping (i.e., for every pair of vectors $(x_1, x_2), (y_1, y_2)$ and scalars c_1, c_2, $F(c_1(x_1, x_2) + c_2(y_1, y_2)) = c_1 F(x_1, x_2) + c_2 F(y_1, y_2)$.) Suppose $F(1, 2) = (2, 3)$ and $F(0, 1) = (1, 4)$. Then in general, $F(x_1, x_2)$ equals

(a) $(x_2, 4x_2)$

(b) $(x_2, x_1 + x_2)$

(c) $(1 + x_1, 4x_2)$

(d) $(x_2, -5x_1 + 4x_2)$

Question 11. A correlation coefficient of 0.2 between Savings and Investment implies that:

(a) A unit change in Income leads to a less than 20 percent increase in Savings

(b) A unit change in Income leads to a 20 percent increase in Savings

(c) A unit change in Income may cause Savings to increase by less than or more than 20

(d) If we plot Savings against Income, the points would lie more or less on a straight line

Question 12. In a simple regression model estimated using OLS, the covariance between the estimated errors and the regressors is zero by construction. This statement is:

(a) True only if the regression model contains an intercept term

(b) True only if the regression model does not contain an intercept term

(c) True irrespective of whether the regression model contains an intercept term

(d) False

Question 13. Consider the uniform distribution over the interval $[a, b]$.

(a) The mean of this distribution depends on the length of the interval, but the variance does not

(b) The mean of this distribution does not depend on the length of the interval, but the variance does

(c) Neither the mean, nor the variance, of this distribution depends on the length of the interval

(d) The mean and the variance of this distribution depend on the length of the interval

The next three questions are based on the following information. Let $F : \Re \to \Re$ be a (cumulative) distribution function. Define $b : [0, 1] \to \Re$ by

$$b(c) = \begin{cases} 0, & \text{if } c = 0 \\ \inf F^{-1}([c, 1]), & \text{if } c \in (0, 1] \end{cases}$$

Question 14. If F has a jump at x, say $c = F(x) > a \geq F(x-)$, then

(a) b has a jump at c

(b) b has a jump at a

(c) b is strictly increasing over (a, c)

(d) b is constant over (a, c)

Question 15. If F is constant over (x, y) with $F(z) < F(x)$ for every $z < x$, then

(a) b has a jump at y

(b) b has a jump at x

(c) b is continuous at $F(x)$

(d) b is decreasing over $[0, F(x)]$

Question 16. Suppose we conduct n independent Bernoulli trials, each with probability of success p. If k is such that the probability of k successes is equal to the probability of $k + 1$ successes, then

(a) $(n + 1)p = n(1 + p)$

(b) $np = (n-1)(1+p)$
(c) np is a positive integer
(d) $(n+1)p$ is a positive integer

The following set of information is relevant for the next 4 questions. Consider a closed economy where at any period t the actual output (Y_t) is demand-determined. Aggregate demand on the other hand has two components: consumption demand (C_t) and investment demand (I_t). Both consumption and investment demands depend on agents' expectation about period t output (Y_t^e) in the following way:

$$\begin{aligned} C_t &= \alpha Y_t^e;\ 0 < \alpha < 1, \\ I_t &= \gamma \left(Y_t^e\right)^2;\ \gamma > 0. \end{aligned}$$

Question 17. Suppose agents have static expectations. Static expectation implies that

(a) in every period agents expect the previous period's actual value to prevail

(b) in every period agents adjust their expected value by a constant positive fraction of the expectational error made in the previous period

(c) in every period agents use all the information available in that period so that the expected value can differ from the actual value if and only if there is a stochastic element present

(d) none of the above

Question 18. Under static expectations, starting from any given initial level of actual output $Y_0 \neq \frac{1-\alpha}{\gamma}$, in the long run the actual output in this economy

(a) will always go to zero
(b) will always go to infinity
(c) will always go to a finite positive value given by $\frac{1-\alpha}{\gamma}$
(d) will go to zero or infinity depending on whether $Y_0 >$ or $< \frac{1-\alpha}{\gamma}$

Question 19. Suppose now agents have rational expectations. Rational expectation implies that

(a) in every period agents expect the previous period's actual value to prevail

(b) in every period agents adjust their expected value by a constant positive fraction of the expectational error made in the previous period

(c) in every period agents use all the information available in that period so that the expected value can differ from the actual value if and only if there is a stochastic element present

(d) none of the above

Question 20. Under rational expectations, in the long run the actual output in this economy

(a) will always go to zero

(b) will always go to infinity

(c) will always go to a finite positive value given by $\frac{1-\alpha}{\gamma}$

(d) will go to zero or infinity depending on agents' expectations

End of Part I.

Proceed to Part II of the examination on the next page.

Part II

- This part of the examination consists of 40 multiple-choice questions. Each question is followed by four possible answers, at least one of which is correct. If more than one choice is correct, choose **only the 'best one'**. The 'best answer' is the one that implies (or includes) the other correct answer(s). Indicate your chosen best answer on the **bubble-sheet** by shading the appropriate bubble.
- For each question, you will get: 2 marks if you choose only the best answer; 0 mark if you choose none of the answers. **However, if you choose something other than the best answer or multiple answers, then you will get $-2/3$ mark for that question**.

The next Two questions are based on the following. Consider a pure exchange economy with three persons, 1, 2, 3, and two goods, x and y. The utilities are given by $u^1(.) = xy$, $u^2(.) = x^3y$ and $u^3(.) = xy^2$, respectively.

Question 21. If the endowments are (2,0), (0,12) and (12,0), respectively, then

(a) an equilibrium price ratio does not exist
(b) $p_X/p_Y = 1$ is an equilibrium price ratio
(c) $p_X/p_Y > 1$ is an equilibrium price ratio
(d) $p_X/p_Y < 1$ is an equilibrium price ratio
Answer: (b)

Question 22. If the endowments are (0,2), (12,0) and (0,12), respectively, then

(a) an equilibrium price ratio does not exist
(b) equilibrium price ratio is the same as in the above question
(c) $p_X/p_Y < 1$ is an equilibrium price ratio
(d) $p_X/p_Y > 1$ is an equilibrium price ratio
Answer: (d)

The next Two questions are based on the following information. A city has a single electricity supplier. Electricity production cost is Rs.

c per unit. There are two types of customers, both have utility function $u_i(q,t) = \theta_i \ln(1+q) - t$, where q is electricity consumption and t is electricity tariff. High type customers are more energy efficient, that is, $\theta_H > \theta_L$; moreover $\theta_L > c$.

Question 23. Suppose the supplier can observe type of the consumer, i.e., whether $\theta = \theta_H$ or $\theta = \theta_L$. If the supplier decides to sell package (q_H, t_H) to those for whom $\theta = \theta_H$ and (q_L, t_L) to those for whom $\theta = \theta_L$, then profit maximizing tariffs will be

(a) $t_H = c \ln\left(\frac{\theta_H}{c}\right)$ and $t_L = c \ln\left(\frac{\theta_L}{c}\right)$
(b) $t_H = \theta_H \ln\left(\frac{\theta_H}{c}\right)$ and $t_L = 0$
(c) $t_H = \theta_H \ln\left(\frac{\theta_H}{c}\right)$ and $t_L = \theta_L \ln\left(\frac{\theta_L}{c}\right)$
(d) $t_H = t_L = c \ln\left(\frac{\theta_H + \theta_L}{c}\right)$
Answer: (c)

Question 24. Now, assume that the supplier cannot observe type of the consumer. Suppose, he puts on offer both of the packages that he would offer in the above question. If consumers are free to choose any of the offered packages, then

(a) Both types will earn zero utility
(b) Only low type can earn positive utility
(c) Only high type can earn positive utility
(d) Both types can earn positive utility
Answer: (c)

Question 25. Suppose buyers of ice-cream are uniformly distributed on the interval $[0, 1]$. Ice-cream sellers 1 and 2 simultaneously locate on the interval, each locating so to maximize her market share given the location of the rival. Each seller's market share corresponds to the proportion of buyers who are located closer to her location than to the rival's location.

(a) Both will locate at 1/2.
(b) One will locate at 1/4 and the other at 3/4.
(c) One will locate at 0 and the other at 1.
(d) One will locate at 1/3 and the other at 2/3.
Answer: (a)

Question 26. In the context of previous question, suppose it is understood by all players that seller 3 will locate on $[0, 1]$ after observing the simultaneous location choices of sellers 1 and 2. Seller 3 aims to maximize market share given the locations of 1 and 2. The locations of sellers 1 and 2 are as follows:

(a) Both will locate at 1/2.
(b) One will locate at 1/4 and the other at 3/4.
(c) One will locate at 0 and the other at 1.
(d) One will locate at 1/3 and the other at 2/3.
Answer: (b)

Question 27. Consider a government and two citizens. The government has to decide whether to create a public good, say a park, at cost Rs 100. The value of the park is Rs 30 to the citizen 1 and Rs 60 to citizen 2; each valuation is private information for the relevant citizen and not known to the government. The government asks the citizens to report their valuations, say r_1 and r_2. It cannot verify the truthfulness of the reports. It decides to build the park if $r_1 + r_2 \geq 100$, in which case, citizen 1 will pay the tax $100 - r_2$ and citizen 2 will pay the tax $100 - r_1$. If the park is not built, then no taxes are imposed. The reported valuations will be

(a) $r_1 < 30$ and $r_2 > 60$
(b) $r_1 > 30$ and $r_2 < 60$
(c) $r_1 = 60$ and $r_2 = 30$
(d) $r_1 = 30$ and $r_2 = 60$
Answer: (d)

Question 28. In the context of the previous question, suppose the only change is that citizen 1's valuation rises to 50 and the same procedure is followed, then

(a) The park will be built and result in a government budget surplus of Rs 10.
(b) The park will be built and result in a government budget deficit of Rs 10.
(c) The park will be built and result in a government balanced budget.
(d) The park will not be built.
Answer: (b)

Question 29. Consider the following two games in which player 1 chooses a row and player 2 chooses a column.

$$\begin{array}{ll} & \text{Hawk} \\ \text{Enter} & \multirow{2}{*}{} \\ \end{array}$$

$$\begin{array}{r} \\ \text{Enter} \\ \text{Not enter} \end{array} \begin{array}{c} \text{Hawk} \\ \begin{pmatrix} -1,1 \\ 0,6 \end{pmatrix} \end{array}$$

$$\begin{array}{r} \\ \text{Enter} \\ \text{Not enter} \end{array} \begin{array}{c} \text{Hawk} \quad \text{Dove} \\ \begin{pmatrix} -1,1 & 3,3 \\ 0,6 & 0,7 \end{pmatrix} \end{array}$$

Analysis of these games shows

(a) Having an extra option cannot hurt.

(b) Having an extra option cannot hurt as long as it dominates other options.

(c) Having an extra option can hurt if the other player is irrational.

(d) Having an extra option can hurt if the other player is rational.

Answer: (d)

Question 30. Consider an exchange economy with agents 1 and 2 and goods x and y. Agent 1 lexicographically prefers x to y. Agent 2's utility function is $\min\{x, y\}$. Agent 1's endowment is $(0, 10)$ and agent 2's endowment is $(10, 0)$. The competitive equilibrium price ratio, p_x/p_y, for this economy

(a) can be any positive number

(b) is greater than 1

(c) is less than 1

(d) does not exist

Answer: (d)

Question 31. Consider a strictly increasing, differentiable function $u : \Re^2 \to \Re$ and the equations:

$$\begin{aligned} \frac{D_1 u(x_1, x_2)}{D_2 u(x_1, x_2)} &= \frac{p_1}{p_2} \text{ and} \\ p_1 x_1 + p_2 x_2 &= w, \end{aligned}$$

where p_1, p_2, w are strictly positive. What additional assumptions will guarantee the existence of continuously differentiable functions $x_1(p_1, p_2, w)$ and $x_2(p_1, p_2, w)$ that will solve these equations for all strictly positive p_1, p_2, w?

(a) u is injective
(b) u is bijective
(c) u is twice continuously differentiable
(d) u is twice continuously differentiable and

$$\begin{bmatrix} \frac{D_{11}u(x_1,x_2)p_2 - D_{12}u(x_1,x_2)p_1}{p_1} & \frac{D_{12}u(x_1,x_2)p_2 - D_{22}u(x_1,x_2)p_1}{p_2} \end{bmatrix} \text{ is nonsingular}$$

Question 32. As $n \uparrow \infty$, the sequence $(-1)^n(1+n^{-1})$
(a) converges to 1
(b) converges to -1
(c) converges to both -1 and 1
(d) does not converge

Question 33. The set $(0,1)$ can be expressed as
(a) the union of a finite family of closed intervals
(b) the intersection of a finite family of closed intervals
(c) the union of an infinite family of closed intervals
(d) the intersection of an infinite family of closed intervals

Question 34. The set $[0,1]$ can be expressed as
(a) the union of a finite family of open intervals
(b) the intersection of a finite family of open intervals
(c) the union of an infinite family of open intervals
(d) the intersection of an infinite family of open intervals

The following information is used in the next two questions. Consider a linear transformation $P : \Re^n \to \Re^n$. Let $\mathcal{R}(P) = \{Px \mid x \in \Re^n\}$ and $\mathcal{N}(P) = \{x \in \Re^n \mid Px = 0\}$.

P is said to be a projector if

(a) every $x \in \Re^n$ can be uniquely written as $x = y + z$ for some $y \in \mathcal{R}(P)$ and $z \in \mathcal{N}(P)$, and

(b) $P(y+z) = y$ for all $y \in \mathcal{R}(P)$ and $z \in \mathcal{N}(P)$.

Question 35. If P is a projector, then
(a) $P^2 = I$, where I is the identity mapping

(b) $P = P^{-1}$
(c) $P^2 = P$
(d) Both (a) and (b)

Question 36. If P is a projector and $Q : \Re^m \to \Re^n$ is a linear transformation such that $\mathcal{R}(P) = \mathcal{R}(Q)$, then
(a) $QP = P$
(b) $PQ = Q$
(c) $QP = I$
(d) $PQ = I$

Question 37. Suppose that a and b are two consecutive roots of a polynomial function f, with $a < b$. Suppose a and b are non-repeated roots. Consequently, $f(x) = (x - a)(x - b)g(x)$ for some polynomial function g. Consider the statements:
I. $g(a)$ and $g(b)$ have opposite signs.
II. $f'(x) = 0$ for some $x \in (a, b)$.
Of these statements,
(a) Both I and II are true.
(b) Only I is true.
(c) Only II is true.
(d) Both I and II are false.

Question 38. Suppose $f : [0, 1] \to \Re$ is a twice differentiable function that satisfies $D^2 f(x) + Df(x) = 1$ for every $x \in (0, 1)$ and $f(0) = 0 = f(1)$. Then,
(a) f does not attain positive values over $(0, 1)$
(b) f does not attain negative values over $(0, 1)$
(c) f attains positive and negative values over $(0, 1)$
(d) f is constant over $(0, 1)$

Question 39. Suppose $x_1, \ldots, x_n$ are positive and $\lambda_1, \ldots, \lambda_n$ are non-negative with $\sum_{i=1}^n \lambda_i = 1$. Then
(a) $\sum_{i=1}^n \lambda_i x_i \geq x_1^{\lambda_1} \ldots x_n^{\lambda n}$
(b) $\sum_{i=1}^n \lambda_i x_i < x_1^{\lambda_1} \ldots x_n^{\lambda n}$
(c) $\sum_{i=1}^n \lambda_i x_i \leq \sqrt{x_1^{\lambda_1} \ldots x_n^{\lambda n}}$
(d) None of the above is necessarily true.

Question 40. Let $\mathcal{N} = \{1, 2, 3, \ldots\}$. Suppose there is a bijection, i.e., a one-to-one correspondence (an "into" and "onto" mapping), between $\mathcal{N}$ and a set X. Suppose there is also a bijection between $\mathcal{N}$ and a set Y. Then,

(a) there is a bijection between $\mathcal{N}$ and $X \cup Y$
(b) there is a bijection between $\mathcal{N}$ and $X \cap Y$
(c) there is no bijection between $\mathcal{N}$ and $X \cap Y$
(d) there is no bijection between $\mathcal{N}$ and $X \cup Y$

The next Three questions pertain to the following: A simple linear regression of wages on gender, run on a sample of 200 individuals, 150 of whom are men, yields the following

$$\begin{aligned} W_i &= 300 - 50 D_i + u_i \\ & \quad\;\; (20) \;\; (10) \end{aligned}$$

where W_i is the wage in Rs per day of the i^{th} individual, $D_i = 1$ if individual i is male, and 0 otherwise, u_i is a classical error term, and the figures in parentheses are standard errors.

Question 41. What is the average wage in the sample?

(a) Rs. 250 per day
(b) Rs. 275 per day
(c) Rs. 262.50 per day
(d) Rs. 267.50 per day

Question 42. The most precise estimate of the difference in wages between men and women would have been obtained if, among these 200 individuals,

(a) There were an equal number (100) of men and women in the sample

(b) The ratio of the number of men and women in the sample was the same as the ratio of their average wages

(c) There were at least 30 men and 30 women; this is sufficient for estimation: precision does not depend on the distribution of the sample across men and women

(d) None of the above

Question 43. The explained (regression) sum of squares in this case is:

(a) 93750

(b) 1406.25
(c) 15000
(d) This cannot be calculated from the information given

Question 44. A researcher estimate the following two models using OLS
Model A: $y_i = \beta_0 + \beta_1 S_i + \beta_2 A_i + \varepsilon_i$
Model B: $y_i = \beta_0 + \beta_1 S_i + \varepsilon_i$
where y_i refers to the marks (out of 100) that a student i gets on an exam, S_i refers to the number of hours spent studying for the exam by the student, and A_i is an index of innate ability (varying continuously from a low ability score of 1 to a high ability score of 10). ε_i the usual classical error term.

The estimated β_1 coefficient is 7.1 for Model A, but 2.1 for Model B; both are statistically significant. The estimated β_2 coefficient is 1.9 and is also significantly different from zero. This suggests that:
(a) Students with lower ability also spend fewer hours studying
(b) Students with lower ability spend more time studying
(c) There is no way that students of even high ability can get more than 40 marks
(d) None of the above

Question 45. An analyst estimates the model $Y = \beta_1 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + u$ using OLS. But the true $\beta_3 = 0$. In this case, by including X_3
(a) there is no harm done as all the estimates would be unbiased and efficient
(b) there is a problem because all the estimates would be biased and inconsistent
(c) the estimates would be unbiased but would have larger standard errors
(d) the estimates may be biased but they would still be efficient

Question 46. Let $\hat{\beta}$ be the OLS estimator of the slope coefficient in a regression of Y on X_1. Let $\tilde{\beta}$ be the OLS estimator of the coefficient on X_1 on a regression of Y on X_1 and X_2. Which of the following is true:
(a) $\text{Var}(\hat{\beta}) < \text{Var}(\tilde{\beta})$
(b) $\text{Var}(\hat{\beta}) > \text{Var}(\tilde{\beta})$
(c) $\text{Var}(\hat{\beta}) <$ or $> \text{Var}(\tilde{\beta})$
(d) $\text{Var}(\hat{\beta}) = \text{Var}(\tilde{\beta})$

Question 47. You estimate the multiple regression $Y = a + b_1X_1 + b_2X_2 + u$ with a large sample. Let $t1$ be the test statistic for testing the null hypothesis $b_1 = 0$ and $t2$ be the test statistic for testing the null hypothesis $b_2 = 0$. Suppose you test the joint null hypothesis that $b_1 = b_2 = 0$ using the principle 'reject the null if either $t1$ or $t2$ exceeds 1.96 in absolute value', taking $t1$ and $t2$ to be independently distributed.

(a) The probability of error Type 1 is 5 percent in this case
(b) The probability of error Type 1 is less than 5 percent in this case
(c) The probability of error Type 1 is more than 5 percent in this case
(d) The probability of error Type 1 is either 5 percent or less than 5 percent in this case

Question 48. Four taste testers are asked to independently rank three different brands of chocolate (A, B, C). The chocolate each tester likes best is given the rank 1, the next 2 and then 3. After this, the assigned ranks for each of the chocolates are summed across the testers. Assume that the testers cannot really discriminate between the chocolates, so that each is assigning her ranks at random. The probability that chocolate A receives a total score of 4 is given by:

(a) $\frac{1}{4}$
(b) $\frac{1}{3}$
(c) $\frac{1}{27}$
(d) $\frac{1}{81}$

Question 49. Suppose 0.1 percent of all people in a town have tuberculosis (TB). A TB test is available but it is not completely accurate. If a person has TB, the test will indicate it with probability 0.999. If the person does not have TB, the test will erroneously indicate that s/he does with probability 0.002. For a randomly selected individual, the test shows that s/he has TB. What is the probability that this person actually has TB?

(a) $\frac{0.002}{0.999}$
(b) $\frac{1}{1000}$
(c) $\frac{1}{3}$
(d) $\frac{2}{3}$

Question 50. There exists a random variable X with mean μ_X and variance σ_X^2 for which $P[\mu_X - 2\sigma_X \leq X \leq \mu_X + 2\sigma_X] = 0.6$. This statement is:

(a) True for any distribution for appropriate choices of μ_X and σ_X^2.

(b) True only for the uniform distribution defined over an appropriate interval

(c) True only for the normal distribution for appropriate choices of μ_X and σ_X^2.

(d) False

Question 51. Consider a sample size of 2 drawn without replacement from an urn containing three balls numbered 1, 2, and 3. Let X be the smaller of the two numbers drawn and Y the larger. The covariance between X and Y is given by:

(a) $\frac{1}{9}$
(b) $\frac{3}{11}$
(c) $\frac{11}{3}$
(d) $\frac{3}{4}$

Question 52. Consider the square with vertices $(0,0)$, $(0,2)$, $(2,0)$ and $(2,2)$. Five points are independently and randomly chosen from the square. If a point (x,y) satisfies $x + 2y \leq 2$, then a pair of dice are rolled. Otherwise, a single die is rolled. Let N be the total number of dice rolled. For $5 \leq n \leq 10$, the probability that $N = n$ is

(a) $\binom{5}{n-5}(1/2)^{n-5}(1/2)^{5-(n-5)}$
(b) $\binom{10}{n-10}(1/4)^{n-10}(3/4)^{n}$
(c) $\binom{5}{n-5}(1/4)^{n-5}(3/4)^{10-n}$
(d) $\binom{10}{n-10}(1/2)^{n-10}(1/2)^{n}$

Question 53. Suppose S is a set with $n > 1$ elements and $A_1, \ldots, A_m$ are subsets of S with the following property: if $x, y \in S$ and $x \neq y$, then there exists $i \in \{1, \ldots, m\}$ such that, either $x \in A_i$ and $y \notin A_i$, or $y \in A_i$ and $x \notin A_i$. Then the following necessarily holds.

(a) $n = 2^m$
(b) $n \leq 2^m$

(c) $n > 2^m$
(d) None of the above

The following set of information is relevant for the next Six questions. Consider the following version of the Solow growth model where the aggregate output at time t depends on the aggregate capital stock (K_t) and aggregate labour force (L_t) in the following way:

$$Y_t = (K_t)^{\alpha} (L_t)^{1-\alpha};\ 0 < \alpha < 1.$$

At every point of time there is full employment of both the factors and each factor is paid its marginal product. Total output is distributed equally to all the households in the form of wage earnings and interest earnings. Households' propensity to save from the two types of earnings differ. In particular, they save s_w proportion of their wage earnings and s_r proportion of their interest earnings in every period. All savings are invested which augments the capital stock over time $\left(\frac{dK}{dt}\right)$. There is no depreciation of capital. The aggregate labour force grows at a constant rate n.

Question 54. Let $s_w = 0$ and $s_r = 1$. An increase in the parameter value α

(a) unambiguously increases the long run steady state value of the capital-labour ratio

(b) unambiguously decreases the long run steady state value of the capital-labour ratio

(c) increases the long run steady state value of the capital-labour ratio if $\alpha > n$

(d) leaves the long run steady state value of the capital-labour ratio unchanged

Question 55. Now suppose $s_r = 0$ and $0 < s_w < 1$. An increase in the parameter value n

(i) unambiguously increases the long run steady state value of the capital-labour ratio

(ii) unambiguously decreases the long run steady state value of the capital-labour ratio

(iii) increases the long run steady state value of the capital-labour ratio if $\alpha > n$

(iv) leaves the long run steady state value of the capital-labour ratio unchanged

Question 56. Now let both s_w and s_r be positive fractions such that $s_w < s_r$. In the long run, the capital-labour ratio in this economy

(a) approaches zero

(b) approaches infinity

(c) approaches a constant value given by $\left[\frac{(1-\alpha)s_w+\alpha s_r}{n}\right]^{\frac{1}{1-\alpha}}$

(d) approaches a constant value given by $\left[\frac{\alpha s_w+(1-\alpha)s_r}{n}\right]^{\frac{1}{\alpha}}$

Question 57. Suppose now the government imposes a proportional tax on wage earnings at the rate τ and redistributes the tax revenue in the form of transfers to the capital-owners. People still save s_w proportion of their net (post-tax) wage earnings and s_r proportion of their net (post-transfer) interest earnings. In the new equilibrium, an increase in the tax rate τ

(a) unambiguously increases the long run steady state value of the capitla-labour ratio

(b) unambiguously decreases the long run steady state value of the capitla-labour ratio

(c) increases the long run steady state value of the capitla-labour ratio if $\alpha > n$

(d) leaves the long run steady state value of the capitla-labour ratio unchanged

Question 58. Let us now go back to case where both s_w and s_r are positive fractions such that $s_w < s_r$ but without the tax-transfer scheme. However, now let the growth rate of labour force be endogenous such that it depends on the economy's capital-labour ratio in the following way:

$$\frac{1}{L_t}\frac{dL}{dt} = \begin{cases} Ak_t \text{ for } k_t < \bar{k}; \\ 0 \text{ for } k_t >> \bar{k}, \end{cases}$$

where $\bar{k} > \left[\frac{(1-\alpha)s_w+\alpha s_r}{A}\right]^{\frac{1}{2-\alpha}}$ is a given constant. In the long run, the capital-labour ratio in this economy

(a) approaches zero

(b) approaches infinity

(c) approaches a constant value given by $\left[\frac{\alpha s_w+(1-\alpha)s_r}{A}\right]^{\frac{1}{\alpha}}$

(d) approaches infinity or a constant value given by $\left[\frac{(1-\alpha)s_w+\alpha s_r}{A}\right]^{\frac{1}{1-\alpha}}$ depending on whether the initial $k_0 >$ or $< \bar{k}$

Question 59. In the above question, an increase in the parameter value A

(a) unambiguously increases the long run steady state value of the capital-labour ratio

(b) unambiguously decreases the long run steady state value of the capital-labour ratio

(c) decreases the long run steady state value of the capital-labour ratio only when the initial $k_0 < \bar{k}$

(d) leaves the long run steady state value of the capital-labour ratio unchanged

Question 60. A profit maximizing firm owns to two production plants with cost functions $c_1(q) = \frac{q^2}{2}$ and $c_2(q) = q^2$, respectively. Firm is free is use either just one or both of the plants, to achieve any given level of output. For this firm, the marginal cost curve

(a) lies above the 45 degree line through the origin, for all positive output levels

(b) lies below the 45 degree line through the origin, for all positive output levels

(c) is the 45 degree line through the origin

(d) none of the above

End of Part II

Rough Work

Rough Work

Rough Work

Rough Work

Rough Work

Rough Work