

**Test code: ME I/ME II, 2005**

**Syllabus for ME I, 2005**

**Matrix Algebra:** Matrices and Vectors, Matrix Operations.

**Permutation and Combination.**

**Calculus:** Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of a single variable, Theory of Sequence and Series.

**Linear Programming:** Formulations, statements of Primal and Dual problems. Graphical Solutions.

**Theory of Polynomial Equations (up to third degree).**

**Elementary Statistics:** Measures of central tendency; dispersion, correlation, Elementary probability theory, Probability mass function, Probability density function and Distribution function.

### Sample Questions for ME I (Mathematics), 2005

For each of the following questions four alternative answers are provided. Choose the answer that you consider to be the most appropriate for a question and write it in your answer book.

- $X \sim B(n, p)$ . The maximum value of  $\text{Var}(X)$  is  
(A)  $\frac{n}{4}$ ; (B)  $n$ ; (C)  $\frac{n}{2}$ ; (D)  $\frac{1}{n}$ .
- The function  $x|x|$  is  
(A) discontinuous at  $x = 0$ ;  
(B) continuous but not differentiable at  $x = 0$ ;  
(C) differentiable at  $x = 0$ ;  
(D) continuous everywhere but not differentiable anywhere.
- If  $A = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$ , then  $A^{100} + A^5$  is  
(A)  $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ ; (B)  $\begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$ ; (C)  $\begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix}$ ; (D) none of these.
- The maximum and minimum values of the function  
 $f(x) = |x^2 + 2x - 3| + 1.5 \log_e x$ , over the interval  $\left[\frac{1}{2}, 4\right]$ , are  
(A)  $(21 + 3 \log_e 2, -1.5 \log_e 2)$ ; (B)  $(21 + \log_e 1.5, 0)$ ;  
(C)  $(21 + 3 \log_e 2, 0)$ ; (D)  $(21 + \log_e 1.5, -1.5 \log_e 2)$ .

5. Let  $\alpha$  and  $\beta$  be the roots of the equation  $x^2 - px + q = 0$ . Define the sequence  $x_n = \alpha^n + \beta^n$ . Then  $x_{n+1}$  is given by

- (A)  $px_n - qx_{n-1}$ ; (B)  $px_n + qx_{n-1}$ ;  
 (C)  $qx_n - px_{n-1}$ ; (D)  $qx_n + px_{n-1}$ .

6. Let  $f : [-1, 1] \rightarrow R$  be twice differentiable at  $x = 0$ ,  $f(0) = f'(0) = 0$ , and  $f''(0) = 4$ . Then the value of  $\lim_{x \rightarrow 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$  is

- (A) 11; (B) 2; (C) 12; (D) none of these.

7. For  $e < x_1 < x_2 < \infty$ ,  $\frac{\log_e x_2}{\log_e x_1}$  is

- (A) less than  $\frac{x_2}{x_1}$ ; (B) greater than  $\frac{x_2}{x_1}$ , but less than  $\left(\frac{x_2}{x_1}\right)^2$ ;  
 (C) greater than  $\left(\frac{x_2}{x_1}\right)^3$ ; (D) greater than  $\left(\frac{x_2}{x_1}\right)^2$ , but less than  $\left(\frac{x_2}{x_1}\right)^3$ .

8. The value of the expression  $\frac{1}{\sqrt{1+\sqrt{2}}} + \frac{1}{\sqrt{2+\sqrt{3}}} + \dots + \frac{1}{\sqrt{99+\sqrt{100}}}$  is

- (A) a rational number lying in the interval (0,9);  
 (B) an irrational number lying in the interval (0,9);  
 (C) a rational number lying in the interval (0,10);  
 (D) an irrational number lying in the interval (0,10).

9. Consider a combination lock consisting of 3 buttons that can be pressed in any combination (including multiple buttons at a time), but in such a way that each number is pressed exactly once. Then the total number of possible combination locks with 3 buttons is

- (A) 6; (B) 9; (C) 10; (D) 13.

10. Suppose the correlation coefficient between  $x$  and  $y$  is denoted by  $R$ , and that between  $x$  and  $(y + x)$ , by  $R_1$ .

- Then, (A)  $R_1 > R$ ; (B)  $R_1 = R$ ;  
(C)  $R_1 < R$  (D) none of these.

11. The value of  $\int_{-1}^1 (x + |x|) dx$  is

- (A) 0; (B) -1; (C) 1; (D) none of these.

12. The values of  $x_1 \geq 0$  and  $x_2 \geq 0$  that maximize  $\Pi = 45x_1 + 55x_2$  subject to  $6x_1 + 4x_2 \leq 120$  and  $3x_1 + 10x_2 \leq 180$

are

- (A) (10,12); (B) (8,5); (C) (12,11); (D) none of the above.

## Syllabus for ME II (Economics), 2005

**Microeconomics:** Theory of consumer behaviour, Theory of producer behaviour, Market forms (Perfect competition, Monopoly, Price Discrimination, Duopoly – Cournot and Bertrand) and Welfare economics.

**Macroeconomics:** National income accounting, Simple model of income determination and Multiplier, IS – LM model, Money, Banking and Inflation.

### Sample questions for ME II (Economics), 2005

1. (a) A divisible cake of size 1 is to be divided among  $n$  ( $>1$ ) persons. It is claimed that the only allocation which is Pareto optimal allocation is  $(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})$ . Do you agree with this claim? Briefly justify your answer.
- (b) Which of the following transactions should be included in GDP? Explain whether the corresponding expenditure is a consumption expenditure or an investment expenditure.
  - (i) Mr. Ramgopal, a private investment banker, hires Mr. Gopi to do cooking and cleaning at home.
  - (ii) Mr. Ramgopal buys a new Maruti Esteem.
  - (iii) Mr. Ramgopal flies to Kolkata from Delhi to see Durga Puja celebration.

- (iv) Mr. Ramgopal directly buys (through the internet) 100 stocks of Satyam Ltd..
- (v) Mr. Ramgopal builds a house.

2. Roses, once in full bloom, have to be picked up and sold on the same day. On any day the market demand function for roses is given by

$$P = \alpha - Q \quad (Q \text{ is number of roses ; } P \text{ is price of a rose}).$$

It is also given that the cost of growing roses, having been incurred by any owner of a rose garden long ago, is *not* a choice variable for him now.

( a ) Suppose, there is only one seller in the market and he finds 1000 roses in full bloom on a day. How many roses should he sell on that day and at what price?

( b ) Suppose there are 10 sellers in the market, and each finds in his garden 100 roses in full bloom ready for sale on a day. What will be the equilibrium price and the number of roses sold on that day? (To answer this part assume  $\alpha \geq 1100$ ).

( c ) Now suppose, the market is served by a large number of price taking sellers. However, the total availability on a day remains unchanged at 1000 roses. Find the competitive price and the total number of roses sold on that day.

3. Laxmi is a poor agricultural worker. Her consumption basket comprises three commodities: rice and two vegetables - *cabbage* and *potato*. But

there are occasionally very hard days when her income is so low that she can afford to buy only rice and no vegetables. However, there never arises a situation when she buys only vegetables and no rice. But when she can afford to buy vegetables, she buys only one vegetable, namely the one that has the lower price per kilogram on that day. Price of each vegetable fluctuates day to day while the price of rice is constant.

Write down a suitable utility function that would represent Laxmi's preference pattern. Explain your answer.

4. Consider a simple Keynesian model for a closed economy without Government. Suppose, saving is proportional to income ( $y$ ), marginal propensity to invest with respect to  $y$  is 0.3 and the system is initially in equilibrium. Now, following a parallel downward shift of the saving function the equilibrium level of saving is found to increase by 12 units. Compute the change in the equilibrium income.

5. Consider an IS-LM model. In the commodity market let the consumption function be given by  $C = a + bY$ ,  $a > 0$ ,  $0 < b < 1$ . Investment and government spending are exogenous and given by  $I_0$  and  $G_0$  respectively. In the money market, the real demand for money is given by  $L = kY - gr$ ,  $k > 0$ ,  $g > 0$ . The nominal money supply and price level are exogenously given at  $M_0$  and  $P_0$  respectively. In these relations  $C$ ,  $Y$  and  $r$  denote consumption, real GDP and interest rate respectively.

- (i) Set up the IS – LM equations.
- (ii) Determine how an increase in the price level  $P_1$ , where  $P_1 > P_0$ , would affect real GDP and the interest rate.