Test code: ME I/ME II, 2007

Syllabus for ME I, 2007

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Linear Programming: Formulations, statements of Primal and Dual problems, Graphical solutions.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory, Probability mass function, Probability density function and Distribution function.

Sample Questions for ME I (Mathematics), 2007

1. Let α and β be any two positive real numbers. Then

   $Lim_{x \to 0} \dfrac{(1+x)^{\alpha}-1}{(1+x)^{\beta}-1}$ equals

   (A) $\dfrac{\alpha}{\beta}$; (B) $\dfrac{\alpha+1}{\beta+1}$; (C) $\dfrac{\alpha-1}{\beta-1}$; (D) 1.

2. Suppose the number X is odd. Then X^2-1 is

   (A) odd; (B) not prime;

   (C) necessarily positive; (D) none of the above.

3. The value of k for which the function $f(x) = ke^{kx}$ is a probability density function on the interval [0, 1] is

(A) $k = \log 2$; (B) $k = 2\log 2$; (C) $k = 3\log 3$; (D) $k = 3\log 4$.

4. p and q are positive integers such that $p^2 - q^2$ is a prime number. Then, $p - q$ is

(A) a prime number; (B) an even number greater than 2;

(C) an odd number greater than 1 but not prime; (D) none of these.

5. Any non-decreasing function defined on the interval $[a, b]$

(A) is differentiable on (a, b);

(B) is continuous in $[a, b]$ but not differentiable;

(C) has a continuous inverse;

(D) none of these.

6. The equation $\begin{vmatrix} x & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 8 & 1 \end{vmatrix} = 0$ is satisfied by

(A) $x = 1$; (B) $x = 3$; (C) $x = 4$; (D) none of these.

7. If $f(x) = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \ldots}}}}$, then $f'(x)$ is

(A) $\dfrac{x}{2f(x) - 1}$; (B) $\dfrac{1}{2f(x) - 1}$; (C) $\dfrac{1}{x\sqrt{f(x)}}$; (D) $\dfrac{1}{2f(x) + 1}$.

8. If $P = \log_x(xy)$ and $Q = \log_y(xy)$, then $P + Q$ equals

(A) PQ; (B) P/Q; (C) Q/P; (D) $(PQ)/2$.

9. The solution to $\int \dfrac{2x^3 + 1}{x^4 + 2x}\, dx$ is

(A) $\dfrac{x^4 + 2x}{4x^3 + 2}$ + constant; (B) $\log x^4 + \log 2x$ + constant;

(C) $\frac{1}{2}\log\left|x^4+2x\right|$ + constant; (D) $\left|\frac{x^4+2x}{4x^3+2}\right|$ + constant.

10. The set of all values of x for which $x^2-3x+2>0$ is

(A) $(-\infty, 1)$; (B) $(2, \infty)$; (C) $(-\infty, 2)\cap(1, \infty)$; (D) $(-\infty, 1)\cup(2, \infty)$.

11. Consider the functions $f_1(x)=x^2$ and $f_2(x)=4x^3+7$ defined on the real line. Then

(A) f_1 is one-to-one and onto, but not f_2;

(B) f_2 is one-to-one and onto, but not f_1;

(C) both f_1 and f_2 are one-to-one and onto;

(D) none of the above.

12. If $f(x)=\left(\frac{a+x}{b+x}\right)^{a+b+2x}$, $a>0,\ b>0$, then $f'(0)$ equals

(A) $\left(\frac{b^2-a^2}{b^2}\right)\left(\frac{a}{b}\right)^{a+b-1}$; (B) $\left(2\log(\frac{a}{b})+\frac{b^2-a^2}{ab}\right)\left(\frac{a}{b}\right)^{a+b}$;

(C) $2\log\left(\frac{a}{b}\right)+\frac{b^2-a^2}{ab}$; (D) $\left(\frac{b^2-a^2}{ba}\right)$.

13. The linear programming problem

$$\max_{x,y}\ z=0.5x+1.5y$$

subject to:

$$x+y\le 6$$
$$3x+y\le 15$$
$$x+3y\le 15$$
$$x,y\ge 0$$

has

(A) no solution; (B) a unique non-degenerate solution;

(C) a corner solution; (D) infinitely many solutions.

14. Let $f(x;\,\theta) = \theta f(x;1) + (1-\theta) f(x;0)$, where θ is a constant satisfying $0 < \theta < 1$. Further, both $f(x;1)$ and $f(x;0)$ are probability density functions (*p.d.f.*). Then

(A) $f(x;\theta)$ is a *p.d.f.* for all values of θ;

(B) $f(x;\theta)$ is a *p.d.f.* only for $0 < \theta < \frac{1}{2}$;

(C) $f(x;\theta)$ is a *p.d.f.* only for $\frac{1}{2} \le \theta < 1$;

(D) $f(x;\theta)$ is not a *p.d.f.* for any value of θ.

15. The correlation coefficient r for the following five pairs of observations

x	5	1	4	3	2
y	0	4	2	0	-1

satisfies

(A) $r > 0$; (B) $r < -0.5$; (C) $-0.5 < r < 0$; (D) $r = 0$.

16. An n-coordinated function f is called homothetic if it can be expressed as an increasing transformation of a homogeneous function of degree one. Let $f_1(x) = \sum_{i=1}^{n} x_i^r$, and $f_2(x) = \sum_{i=1}^{n} a_i x_i + b$, where $x_i > 0$ for all *i*, $0 < r < 1$, $a_i > 0$ and *b* are constants. Then

(A) f_1 is not homothetic but f_2 is; (B) f_2 is not homothetic but f_1 is;

(C) both f_1 and f_2 are homothetic; (D) none of the above.

17. If $h(x) = \frac{1}{1-x}$, then $h(h(h(x)))$ equals

(A) $\dfrac{1}{1-x}$; (B) x; (C) $\dfrac{1}{x}$; (D) 1- x.

18. The function $x|x| + \left(\dfrac{|x|}{x}\right)^3$ is

(A) continuous but not differentiable at $x = 0$;

(B) differentiable at $x = 0$;

(C) not continuous at $x = 0$;

(D) continuously differentiable at $x = 0$.

19. $\displaystyle\int \frac{2dx}{(x-2)(x-1)x}$ equals

(A) $\log\left|\dfrac{x(x-2)}{(x-1)^2}\right|$ + constant;

(B) $\log\left|\dfrac{(x-2)}{x(x-1)^2}\right|$ + constant;

(C) $\log\left|\dfrac{x^2}{(x-1)(x-2)}\right|$ + constant;

(D) $\log\left|\dfrac{(x-2)^2}{x(x-1)}\right|$ + constant.

20. Experience shows that 20% of the people reserving tables at a certain restaurant never show up. If the restaurant has 50 tables and takes 52 reservations, then the probability that it will be able to accommodate everyone is

(A) $1-\dfrac{209}{552}$; (B) $1-14\times\left(\dfrac{4}{5}\right)^{52}$; (C) $\left(\dfrac{4}{5}\right)^{50}$; (D) $\left(\dfrac{1}{5}\right)^{50}$.

21. For any real number *x*, define [*x*] as the highest integer value not greater than *x*. For example, [0.5] = 0, [1] = 1 and [1.5] = 1. Let $I = \int_0^{3/2} \{[x] + [x^2]\} dx$. Then *I* equals

(A) 1; (B) $\frac{5-2\sqrt{2}}{2}$;

(C) $2\sqrt{2}$; (D) none of these.

22. Every integer of the form $(n^3 - n)(n^2 - 4)$ (for $n = 3, 4, \ldots$) is

(A) divisible by 6 but not always divisible by 12;

(B) divisible by 12 but not always divisible by 24;

(C) divisible by 24 but not always divisible by 120;

(D) divisible by 120 but not always divisible by 720.

23. Two varieties of mango, A and B, are available at prices Rs. p_1 and Rs. p_2 per kg, respectively. One buyer buys 5 kg. of A and 10 kg. of B and another buyer spends Rs. 100 on A and Rs. 150 on B. If the average expenditure per mango (irrespective of variety) is the same for the two buyers, then which of the following statements is the most appropriate?

(A) $p_1 = p_2$; (B) $p_2 = \frac{3}{4} p_1$;

(C) $p_1 = p_2$ or $p_2 = \frac{3}{4} p_1$; (D) $\frac{3}{4} \le \frac{p_2}{p_1} < 1$.

24. For a given bivariate data set $(x_i, y_i;\ i = 1, 2, \ldots, n)$, the squared correlation coefficient (r^2) between x^2 and y is found to be 1. Which of the following statements is the most appropriate?

(A) In the (x, y) scatter diagram, all points lie on a straight line.

(B) In the (x, y) scatter diagram, all points lie on the curve $y = x^2$.

(C) In the (x, y) scatter diagram, all points lie on the curve $y = a + bx^2, a > 0, b > 0$.

(D) In the (x, y) scatter diagram, all points lie on the curve $y = a + bx^2$, a, b any real numbers.

25. The number of possible permutations of the integers 1 to 7 such that the numbers 1 and 2 always precede the number 3 and the numbers 6 and 7 always succeed the number 3 is

(A) 720; (B) 168;

(C) 84; (D) none of these.

26. Suppose the real valued continuous function f defined on the set of non-negative real numbers satisfies the condition $f(x) = xf(x)$, then $f(2)$ equals

(A) 1; (B) 2;

(C) 3; (D) $f(1)$.

27. Suppose a discrete random variable X takes on the values $0, 1, 2, \ldots, n$ with frequencies proportional to binomial coefficients $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$ respectively. Then the mean (μ) and the variance (σ^2) of the distribution are

(A) $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{2}$;

(B) $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{4}$;

(C) $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{4}$;

(D) $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{2}$.

28. Consider a square that has sides of length 2 units. *Five* points are placed anywhere inside this square. Which of the following statements is **incorrect**?

(A) There cannot be any two points whose distance is more than $2\sqrt{2}$.

(B) The square can be partitioned into four squares of side 1 unit each such that at least one unit square has two points that lies on or inside it.

(C) At least two points can be found whose distance is less than $\sqrt{2}$.

(D) Statements (A), (B) and (C) are all incorrect.

29. Given that *f* is a real-valued differentiable function such that $f(x)f'(x) < 0$ for all real *x*, it follows that

(A) $f(x)$ is an increasing function;

(B) $f(x)$ is a decreasing function;

(C) $|f(x)|$ is an increasing function;

(D) $|f(x)|$ is a decreasing function.

30. Let p, q, r, s be four arbitrary positive numbers. Then the value of $\frac{(p^2+p+1)(q^2+q+1)(r^2+r+1)(s^2+s+1)}{pqrs}$ is at least as large as

(A) 81;

(B) 91;

(C) 101.

(D) None of these.

Syllabus for ME II (Economics), 2007

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2007

1. (a) There is a cake of size 1 to be divided between two persons, 1 and 2. Person 1 is going to cut the cake into two pieces, but person 2 will select one of the two pieces for himself first. The remaining piece will go to person 1. What is the optimal cutting decision for player 1? Justify your answer.

(b) Kamal has been given a free ticket to attend a classical music concert. If Kamal had to pay for the ticket, he would have paid up to Rs. 300/- to attend the concert. On the same evening, Kamal's alternative entertainment option is a film music and dance event for which tickets are priced at Rs. 200/- each. Suppose also that Kamal is willing to pay up to Rs. X to attend the film music and dance event. What does Kamal do, i.e., does he attend the classical music concert, or does he attend the film music and dance show, or does he do neither? Justify your answer.

2. Suppose market demand is described by the equation P = 300 – Q and competitive conditions prevail. The short-run supply curve is P = -180 +5Q. Find the initial short-run equilibrium price and quantity. Let the long-run supply curve be P = 60 + 2Q. Verify whether the market is also in the long-run equilibrium at the initial short-run equilibrium that you have worked out. Now suppose that the market demand at every price is

doubled. What is the new market demand curve? What happens to the equilibrium in the *very* short-run? What is the new short-run equilibrium? What is the new long-run equilibrium? If a price ceiling is imposed at the old equilibrium, estimate the perceived shortage. Show all your results in a diagram.

3. (a) Suppose in year 1 economic activities in a country constitute only production of wheat worth Rs. 750. Of this, wheat worth Rs. 150 is exported and the rest remains unsold. Suppose further that in year 2 no production takes place, but the unsold wheat of year 1 is sold domestically and residents of the country import shirts worth Rs. 250. Fill in, with adequate explanation, the following chart :

Year	GDP	= Consumption	+ Investment	+ Export	- Import
1	____	____	____	____	____
2	____	____	____	____	____

(b) Consider an IS-LM model for a closed economy with government where investment (I) is a function of rate of interest (r) only. An increase in government expenditure is found to crowd out 50 units of private investment. The government wants to prevent this by a minimum change in the supply of real money balance. It is given that $\frac{dI}{dr} = -50$, slope of the LM curve, $\frac{dr}{dy}(LM) = \frac{1}{250}$, slope of the IS curve, $\frac{dr}{dy}(IS) = -\frac{1}{125}$, and all relations are linear. Compute the change in y from the initial to the final equilibrium when all adjustments have been made.

4. (a) Consider a consumer with income W who consumes *three* goods, which we denote as $i = 1, 2, 3$. Let the amount of good i that the consumer consumes be x_i and the price

of good i be p_i. Suppose that the consumer's preference is described by the utility function $U(x_1, x_2, x_3) = x_1\sqrt{x_2 x_3}$.

(i) Set up the utility maximization problem and write down the Lagrangian.

(ii) Write down the first order necessary conditions for an interior maximum and then obtain the Marshallian (or uncompensated) demand functions.

(b) The production function, $Y = F(K, L)$, satisfies the following properties: (i) CRS, (ii) symmetric in terms of inputs and (iii) $F(1,1) = 1$. The price of each input is Rs. 2/- per unit and the price of the product is Rs. 3/- per unit. *Without using calculus* find the firm's optimal level of production.

5.(a) A monopolist has contracted to sell as much of his output as he likes to the government at Rs.100/- per unit. His sale to the government is positive. He also sells to private buyers at Rs 150/- per unit. What is the price elasticity of demand for the monopolist's products in the private market?

(b) Mrs. Pathak is very particular about her consumption of tea. She always takes 50 grams of sugar with 20 grams of ground tea. She has allocated Rs 55 for her spending on tea and sugar per month. (Assume that she doesn't offer tea to her guests or anybody else and she doesn't consume sugar for any other purpose). Sugar and tea are sold at 2 paisa per 10 grams and 50 paisa per 10 grams respectively. Determine how much of tea and sugar she demands per month.

(c) Consider the IS-LM model with government expenditure and taxation. A change in the income tax rate changes the equilibrium from $(y = 3000,\ r = 4\%)$ to $(y = 3500,\ r = 6\%)$, where y, r denote income and rate of interest, respectively. It is given that a unit increase in y increases demand for real money balance by 0.25 of a unit. Compute the change in real money demand that results from a 1% increase in the rate of interest. (Assume that all relationships are linear.)

6. (a) An economy produces two goods, corn and machine, using for their production only labor and some of the goods themselves. Production of one unit of corn requires 0.1 units of corn, 0.3 machines and 5 man-hours of labor. Similarly, production of one machine requires 0.4 units of corn, 0.6 machines and 20 man-hours of labor.

(i) If the economy requires 48 units of corn but no machine for final consumption, how much of each of the two commodities is to be produced? How much labor will be required?

(ii) If the wage rate is Rs. 2/- per man-hour, what are the prices of corn and machines, if price of each commodity is equated to its average cost of production?

(b) Consider two consumers A and B, each with income W. They spend their entire budget over the two commodities, X and Y. Compare the demand curves of the two consumers under the assumption that their utility functions are $U_A = x + y$ and $U_B = x^2 + y^2$ respectively.

7. Consider a Simple Keynesian Model without government for an open economy, where both consumption and import are proportional functions of income (Y). Suppose that average propensities to consume and import are 0.8 and 0.3, respectively. The investment (I) function and the level of export (X) are given by $I = 100 + 0.4Y$ and $X = 100$.

(i) Compute the aggregate demand function if the maximum possible level of imports is 450. Can there be an equilibrium for this model? Show your result graphically.

(ii) How does your answer to part (i) change if the limit to import is raised to 615? What can you say about the stability of equilibrium if it exists?

8. Suppose an economic agent's life is divided into two periods, the first period constitutes her youth and the second her old age. There is a single consumption good, C, available in both periods and the agent's utility function is given by

$$u(C_1, C_2) = \frac{C_1^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\frac{C_2^{1-\theta} - 1}{1-\theta}, \qquad 0 < \theta < 1, \rho > 0,$$

where the first term represents utility from consumption during youth. The second term represents discounted utility from consumption in old age, $1/(1+\rho)$ being the discount factor. During the period, the agent has a unit of labour which she supplies inelastically for a wage rate w. Any savings (i.e., income minus consumption during the first period) earns a rate of interest r, the proceeds from which are available in old age in units of the only consumption good available in the economy. Denote savings by s. The agent maximizes utility subjects to her budget constraint.

i) Show that θ represents the elasticity of marginal utility with respect to consumption in each period.

ii) Write down the agent's optimization problem, i.e., her problem of maximizing utility subject to the budget constraint.

iii) Find an expression for s as a function of w and r.

(iv)How does s change in response to a change in r? In particular, show that this change depends on whether θ exceeds or falls short of unity.

(v)Give an intuitive explanation of your finding in (iv)

9. Consider a neo-classical one-sector growth model with the production function $Y = \sqrt{KL}$. If 30% of income is invested and capital stock depreciates at the rate of 7% and labour force grows at the rate of 3%, find out the level of per capita income in the steady-state equilibrium.