SYLLABUS & SAMPLE QUESTIONS FOR MS (QE)

2011

Syllabus for ME I, 2011

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Algebra: Binomial Theorem, AP, GP, HP, exponential, logarithmic series.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation and regression, Elementary probability theory, Probability distributions.

Sample Questions for ME I (Mathematics), 2011

1. The expression $\sqrt{13+3\sqrt{23/3}}+\sqrt{13-3\sqrt{23/3}}$ is

   (a) A natural number,
   (b) A rational number but not a natural number,
   (c) An irrational number not exceeding 6,
   (d) An irrational number exceeding 6.

2. The domain of definition of the function $f(x)=\frac{\sqrt{(x+3)}}{(x^2+5x+4)}$ is

   (a) $(-\infty,\infty)\setminus\{-1,-4\}$,
   (b) $(-0,\infty)\setminus\{-1,-4\}$
   (c) $(-1,\infty)\setminus\{-4\}$
   (d) None of these.

3. The value of

$$\log_4 2 - \log_8 2 + \log_{16} 2 - \ldots\ldots.$$

(a) $\log_e 2$, (b) $1-\log_e 2$,
(c) $\log_e 2 - 1$, (d) None of these.

4. The function $\max\{1, x, x^2\}$, where x is any real number, has

(a) Discontinuity at one point only,
(b) Discontinuity at two points only,
(c) Discontinuity at three points only,
(d) No point of discontinuity.

5. If $x, y, z > 0$ are in HP, then $\frac{x-y}{y-z}$ equals

(a) $\frac{x}{y}$, (b) $\frac{y}{z}$, (c) $\frac{x}{z}$, (d) None of these.

6. The function $f(x) = \frac{x}{1+|x|}$, where x is any real number is,

(a) Everywhere differentiable but the derivative has a point of discontinuity.
(b) Everywhere differentiable except at 0.
(c) Everywhere continuously differentiable.
(d) Everywhere differentiable but the derivative has 2 points of discontinuity.

7. Let the function $f: R_{++} \to R_{++}$ be such that $f(1) = 3$ and $f'(1) = 9$, where R_{++} is the positive part of the real line. Then $\lim_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ equals

(a) 3, (b) e^2, (c) 2, (d) e^3.

8. Let $f, g: [0,\infty) \to [0,\infty)$ be decreasing and increasing respectively. Define $h(x) = f(g(x))$. If $h(0) = 0$, then $h(x) - h(1)$ is

(a) Nonpositive for $x \geq 1$, positive otherwise, (b) Always negative,
(c) Always positive, (d) Positive for $x \geq 1$, nonpositive otherwise.

9. A committee consisting of 3 men and 2 women is to be formed out of 6 men and 4 women. In how many ways this can be done if Mr. X and Mrs. Y are not to be included together?

(a) 120, (b) 140, (c) 90, (d) 60.

10. The number of continuous functions f satisfying $x\,f(y) + y\,f(x) = (x+y)\,f(x)\quad f(y)$, where x and y are any real numbers, is

(a) 1, (b) 2, (c), 3,
(d) None of these.

11. If the positive numbers $x_1, \ldots, x_n$ are in AP, then

$$\frac{1}{\sqrt{x_1}+\sqrt{x_2}} + \frac{1}{\sqrt{x_2}+\sqrt{x_3}} + \ldots\ldots + \frac{1}{\sqrt{x_{n-1}}+\sqrt{x_n}} \text{ equals}$$

(a) $\dfrac{n}{\sqrt{x_1}+\sqrt{x_n}}$, (b) $\dfrac{1}{\sqrt{x_1}+\sqrt{x_n}}$,

(b) $\dfrac{2n}{\sqrt{x_1}+\sqrt{x_n}}$, (d) None of these.

12. If x, y, z are any real numbers, then which of the following is always true?

(a) $\max\{x, y\} < \max\{x, y, z\}$,
(b) $\max\{x, y\} > \max\{x, y, z\}$,

(c) $\max\{x, y\} = \dfrac{x + y + |x - y|}{2}$

(d) None of these.

13. If $x_1, x_2, x_3, x_4 > 0$ and $\sum_{i=1}^{4} x_i = 2$, then $P = (x_1 + x_2)(x_3 + x_4)$ is

(a) Bounded between zero and one,
(b) Bounded between one and two,
(c) Bounded between two and three,
(d) Bounded between three and four.

14. Everybody in a room shakes hand with everybody else. Total number of handshakes is 91. Then the number of persons in the room is

(a) 11, (b) 12, (c) 13,
(d) 14.

15. The number of ways in which 6 pencils can be distributed between two boys such that each boy gets at least one pencil is

(a) 30, (b) 60, (c) 62, (d) 64.

16. Number of continuous functions characterized by the equation $x f(x) + 2f(-x) = -1$, where x is any real number, is

(a) 1, (b) 2, (c) 3, (d) None of these.

17. The value of the function $f(x) = x + \int_0^1 (xy^2 + x^2 y) f(y) dy$ is

$px + qx^2$, where
(a) p = 80, q = 180,
(b) p = 40, q = 140
(c) p = 50, q = 150,
(d) None of these.

18. If x and y are real numbers such that $x^2 + y^2 = 1$, then the maximum value of $|x| + |y|$ is

(a) $\frac{1}{2}$, (b) $\sqrt{2}$, (c) $\frac{1}{\sqrt{2}}$, (d) 2.

19. The number of onto functions from $A=\{p,q,r,s\}$ to $B=\{p,r\}$ is

(a) 16, (b) 2, (c) 8, (d) 14.

20. If the coefficients of $(2r+5)$ th and $(r-6)$ th terms in the expansion of $(1+x)^{39}$ are equal, then ${}^{r}C_{12}$ equals

(a) 45, (b) 91, (c) 63, (d) None of these.

21. If $X=\begin{bmatrix} C & 2 \\ 1 & C \end{bmatrix}$ and $|X^7|=128$, then the value of C is

(a) ± 5, (b) ± 1, (c) ± 2, (d) None of these.

22. Let $f(x)=Ax^2+Bx+C$, where A, B, C re real numbers. If $f(x)$ is an integer whenever x is an integer, then

(a) $2A$ and $A+B$ are integers, but C is not an integer.
(b) $A+B$ and C are integers, but $2A$ is not an integer.
(c) $2A$, $A+B$ and C are all integers.
(d) None of these.

23. Four persons board a lift on the ground floor of a seven-storey building. The number of ways in which they leave the lift, such that each of them gets down at different floors, is

(a) 360, (b) 60, (c) 120, (d) 240.

24. The number of vectors (x,x_1,x_2), where $x,x_1,x_2>0$, for which

$$|\log(xx_1)|+|\log(xx_2)|+\left|\log\left(\frac{x}{x_1}\right)\right|+\left|\log\left(\frac{x}{x_2}\right)\right|$$

$= |\log x_1+\log x_2|$ holds, is

(a) One, (b) Two, (c) Three, (d) None of these.

25. In a sample of households actually invaded by small pox, 70% of the inhabitants are attacked and 85% had been vaccinated. The minimum percentage of households (among those vaccinated) that

must have been attacked [Numbers expressed as nearest integer value] is

(a) 55, (b) 65, (c) 30, (d) 15.

26. In an analysis of bivariate data (X and Y) the following results were obtained.

Variance of $X(\sigma_x^2) = 9$, product of the regression coefficient of Y on X and X on Y is 0.36, and the regression coefficient from the regression of Y on X (β_{yx}) is 0.8.

The variance of Y is

(a) 16, (b) 4, (c) 1.69 ,d) 3.

27. For comparing the wear and tear quality of two brands of automobile tyres, two samples of 50 customers using two types of tyres under similar conditions were selected. The number of kilometers x_1 and x_2 until the tyres became worn out, was obtained from each of them for the tyres used by them. The sample results were as follows : $\bar{x}_1 = 13{,}200$ km, $\bar{x}_2 = 13{,}650$ km, S_{x1} = 300 km, S_{x2} = 400 km. What would you conclude about the two brands of tyres (at 5% level of significance) as far as the wear and tear quality is concerned?

(a) The two brands are alike ,
(b) The two brands are not the same,
(c) Nothing can be concluded,
(d) The given data are inadequate to perform a test.

28. A continuous random variable x has the following probability density function:

$$f(x) = \frac{\alpha}{x_0}\left(\frac{x_0}{x}\right)^{\alpha+1} \quad \text{for } x > x_0,\ \alpha > 1.$$

The distribution function and the mean of x are given respectively by

(a) $1 - \left(\frac{x}{x_0}\right)^{\alpha}, \frac{\alpha - 1}{\alpha} x_0$,

(b) $1-\left(\frac{x}{x_0}\right)^{-\alpha}, \frac{\alpha-1}{\alpha}x_0,$

(c) $1-\left(\frac{x}{x_0}\right)^{-\alpha}, \frac{\alpha x_0}{\alpha-1},$

(d) $1-\left(\frac{x}{x_0}\right)^{\alpha}, \frac{\alpha x_0}{\alpha-1}$

29. Suppose a discrete random variable X takes on the values $0, 1, 2, \ldots, n$ with frequencies proportional to binomial coefficients $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$ respectively. Then the mean (μ) and the variance (σ^2) of the distribution are

(A) $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{2}$;

(B) $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{4}$;

(C) $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{4}$;

(D) $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{2}$.

30. Let $\{X_i\}$ be a sequence of $i.i.d$ random variables such that

$X_i = 1$ with probability p

$= 0$ with probability $1 - p$

Define $y = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} X_i = 100 \\ 0 & \text{otherwise} \end{cases}$

Then $E(y^2)$ is

(a) ∞, (b) $\binom{n}{100} p^{100}(1-p)^{n-100}$, (c) np, (d) $(np)^2$.

Syllabus for ME II (Economics), 2011

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2011

1. A monopolist sells two products, X and Y. There are three consumers with asymmetric preferences. Each consumer buys either one unit of a product or does not buy the product at all. The per-unit maximum willingness to pay of the consumers is given in the table below.

Consumer No.	X	Y
1	4	0
2	3	3
3	0	4

The monopolist who wants to maximize total payoffs has three alternative marketing strategies: (i) sell each commodity separately and so charge a uniform unit price for each commodity separately (simple monopoly pricing); (ii) offer the two commodities for sale only in a package comprising of one unit of each, and hence charge a price for the whole bundle (pure bundling strategy), and (iii) offer each commodity separately as well as a package of both, that is, offer unit price for each commodity as well as charge a bundle price (mixed bundling strategy). However, the monopolist cannot price discriminate between the consumers. Given the above data, find out the monopolist's optimal strategy and the corresponding prices of the products.

[30]

2. Consider two consumers with identical income M and utility function $U = xy$ where x is the amount of restaurant good consumed and y is the amount of any other good consumed. The unit prices of the goods are given. The consumers have two alternative plans to meet the restaurant bill. Plan A: they eat together at the restaurant and each pays his own bill. Plan B: they eat together at the restaurant but each pays one-half of the total restaurant bill. Find equilibrium consumption under plan A.

   (a) Find equilibrium consumption under plan B.

   (b) Explain your answer if the equilibrium outcome in case (b) differs from that in case (a).

[6+18+6=30]

3. Consider a community having a fixed stock X of an exhaustible resource (like oil) and choosing, over an infinite horizon, how much of this resource is to be used up each period. While doing so, the community maximizes an intertemporal utility function $U = \sum_{t=0}^{\infty} \delta^t \ln C_t$ where C_t represents consumption or use of the resource at period t and δ $(0 < \delta < 1)$ is the discount factor.

(a) Set up the utility maximization problem of the community and interpret the first order condition.

(b) Express the optimal consumption C_t for any period t in terms of the parameters δ and X.

(c) If an unanticipated discovery of an additional stock of X' occurs at the beginning of period T $(0 < T < \infty)$, what will be the new level of consumption at each period from T onwards?

[7+16+7=30]

4. A consumer, with a given money income M, consumes n goods $x_1, x_2, ..., x_n$ with given prices $p_1, p_2, ..., p_n$.

(a) Suppose his utility function is $U(x_1, x_2, ..., x_n) = Max(x_1, x_2, ..., x_n)$. Find the Marshallian demand function for good x_i and draw it in a graph.

(b) Suppose his utility function is $U(x_1, x_2, ..., x_n) = Min(x_1, x_2, ..., x_n)$. Find the income and the own price elasticities of demand for good x_i.

[15+15=30]

5. An economy, consisting of m individuals, is endowed with quantities $\omega_1, \omega_2, ...\omega_n$ of n goods. The ith individual has a utility function $U(C_1^i, C_2^i, ...C_n^i) = C_1^i C_2^i ... C_n^i$, where C_j^i is consumption of good j of individual i.

(a) Define an *allocation*, a *Pareto inferior allocation* and a *Pareto optimal allocation* for this economy.

(b) Find an allocation which is *Pareto inferior* and an allocation which is *Pareto optimal.*

(c) Consider an allocation where $C_j^i = \lambda^i \omega_j \forall j$, $\sum_i \lambda^i = 1$. Is this allocation *Pareto optimal*?

[6+18+6=30]

6. Suppose that a monopolist operates in a domestic market facing a demand curve $p = 5 - \frac{3}{2} q_h$, where p is the domestic price and q_h is the quantity sold in the domestic market. This monopolist also has the option of selling the product in the foreign market at a constant price of 3. The monopolist has a cost function given by $C(q) = q^2$, where q is the total quantity that the monopolist produces. Now, answer the following questions.

(a) How much will the monopolist sell in the domestic market and how much will it sell in the foreign market?

(b) Suppose, the home government imposes a restriction on the amount that the monopolist can sell in the foreign market. In particular, the monopolist is not allowed to sell more than 1/6 units of the good in the foreign market. Now find out the amount the monopolist sells in the domestic market and in the foreign market.

[6+24=30]

7. An economy produces two goods, food (F) and manufacturing (M).

Food is produced by the production function $F = (L_F)^{\frac{1}{2}}(T)^{\frac{1}{2}}$, where L_F is the labour employed, T is the amount of land used and F is the amount of food produced. Manufacturing is produced by the production function $M = (L_M)^{\frac{1}{2}}(K)^{\frac{1}{2}}$, where L_M is the labour employed, K is the amount of capital used and M is the amount of manufacturing production. Labour is perfectly mobile between the sectors (i.e. food and manufacturing production) and the total amount of labour in this economy is denoted by L. All the factors of production are fully employed. Land is owned by the landlords and capital is owned by the capitalists. You are also provided with the following data: $K = 36$, $T = 49$, and $L = 100$. Also assume that the price of food and that of manufacturing are the same and is equal to unity.

(a) Find out the equilibrium levels of labour employment in the food sector and the manufacturing sector (i.e. L_F and L_M respectively)

(b) Next, we introduce a small change in the description of the economy given above. Assume, everything remains the same except for the fact that land is owned by none; land comes for free! How much labour would now be employed in the food and the manufacturing sectors?

(c) Suggest a measure of welfare for the economy as a whole.

(d) Using the above given data and your measure of welfare, determine whether the scenario given in problem (b), where land is owned by none, better or worse for the economy as a whole, compared to the scenario given in problem (a), where land is owned by the landlords?

(e) What do you think is the source of the difference in welfare levels (if any) under case (a) and case (b).

[6+10+4+6+4=30]

8. An economy produces a single homogeneous good in a perfectly competitive set up, using the production function $Y = AF(L, K)$, where Y is the output of the good, L and K are the amount of labour and capital respectively and A is the technological productivity parameter. Further, assume that F is homogeneous of degree one in L and K. Labour and capital in this economy remains fully employed. It has also been observed that the total wage earning of this economy is equal to the total earnings of capital in the economy at all points in time.

Answer the following questions.

(a) It is observed that over a given period the labour force grew by 4%, the capital stock grew by 3%, and output grew by 9%. What then was the growth rate of the technological productivity parameter (A) over that period?

(b) Over another period the wage rate of labour in this economy exhibited a growth of 30% , rental rate of capital grew by 10% and the price of the good over the same period grew by 5%. Find out the growth rate of the technological productivity parameter (A) over this period.

(c) Over yet another period, it was observed that there was no growth in the technological productivity, and the wages grew by 30% and rental rate grew by 10%. Infer from this, the growth rate of the price of the good over the period.

[4+20+6=30]

9. An economy produces two goods - m and g. Capitalists earn a total income, R (R_m from sector m plus R_g from sector g), but consumes only good m, spending a fixed proportion (c) of their income on it. Workers do *not* save. Workers in sector m spend a fixed proportion α of their income (W) on good g and the rest on good m. [However, whatever wages are paid in sector g are spent entirely for the consumption of good g only so that we ignore wages in this sector for computing both income generation therein and the expenditure made on its output.] The categories of income and expenditure in the two sectors are shown in detail in the chart below.

Sector *m*		Sector *g*	
Income generated	Expenditure on good *m*	Income (net of wages) generated	Expenditure (net of that by own workers) on good g
• Capitalists' income (R_m) • Wages (W)	• Capitalists' consumption (C = c. R) • Consumption of workers of sector *m*: ($\{1-\alpha\}.W$) • Investment: (I)	Capitalists' Income (R_g)	• Consumption of workers of sector *m* (α .W)

Further, *investment* expenditure (I), made exclusively on *m*-good, is *autonomous* and income distribution in sector *m* is exogenously given:

$R_m = \theta . W$ (θ given).

(a) Equating aggregate income with aggregate expenditure for the economy, show that capitalists' income (R) is determined exclusively by their *own* expenditure (C and I). Is there any multiplier effect of I on R ? Give arguments.

(b) Show that I (along with c, α and θ) also determines W.

[15 +15= 30]

10. Consider two countries – a domestic country (with excess capacity and unlimited supply of labour) and a benevolent foreign country. The domestic country produces a single good at a fixed price of Re.1 per

unit and is in equilibrium initially (i.e. in year 0) with income at Rs. 100 and consumption, investment and savings at Rs. 50 each. Investment expenditure is autonomous. Final expenditure in any year t shows up as income in year t (Y_t), but consumption expenditure in year t (C_t) is given by: $C_t = 0.5\ Y_{t-1}$. The foreign country agrees to give a loan of Rs.100 to the domestic country in year 1 at *zero* interest rate, but on conditions that it be (i) used for investment only and (ii) repaid in full at the beginning of the next year. The loan may be renewed every year, but on the same conditions as above. Find out income, consumption, investment and savings of the domestic country in year 1, year 2 and in final equilibrium in each of the following two alternative cases:

(a) The country takes the loan in year 1 only.

(b) The country takes the loan in year 1 and renews it every year.

[15 + 15=30]