SYLLABUS AND SAMPLE QUESTIONS FOR MS(QE)

2012

Syllabus for ME I (Mathematics), 2012

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations (up to third degree).

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Elementary Statistics: Elementary probability theory, measures of central tendency; dispersion, correlation and regression, probability distributions, standard distributions–Binomial and Normal.

Sample Questions for MEI (Mathematics), 2012

1. Kupamonduk, the frog, lives in a well 14 feet deep. One fine morning she has an urge to see the world, and starts to climb out of her well. Every day she climbs up by 5 feet when there is light, but slides back by 3 feet in the dark. How many days will she take to climb out of the well?
(A) 3,
(B) 8,
(C) 6,
(D) None of the above.

2. The derivative of $f(x) = |x|^2$ at $x = 0$ is,
(A) -1,
(B) Non-existent,
(C) 0,
(D) 1/2.

3. Let $\mathcal{N} = \{1, 2, 3, \ldots\}$ be the set of natural numbers. For each $n \in \mathcal{N}$, define $A_n = \{(n+1)k : k \in \mathcal{N}\}$. Then $A_1 \cap A_2$ equals
(A) A_3,
(B) A_4,
(C) A_5,
(D) A_6.

4. Let $S = \{a, b, c\}$ be a set such that a, b and c are distinct real numbers. Then $\min\{\max\{a, b\}, \max\{b, c\}, \max\{c, a\}\}$ is always
(A) the highest number in S,
(B) the second highest number in S,
(C) the lowest number in S,
(D) the arithmetic mean of the three numbers in S.

5. The sequence $< -4^{-n} >$, $n = 1, 2, \cdots$, is
(A) Unbounded and monotone increasing,
(B) Unbounded and monotone decreasing,
(C) Bounded and convergent,
(D) Bounded but not convergent.

6. $\int \frac{x}{7x^2+2} dx$ equals

(A) $\frac{1}{14} ln(7x^2 + 2) +$ constant,
(B) $7x^2 + 2$,
(C) $ln\ x+$ constant,
(D) None of the above.

7. The number of real roots of the equation

$$2(x-1)^2 = (x-3)^2 + (x+1)^2 - 8$$

is
(A) Zero,
(B) One,
(C) Two,
(D) None of the above.

8. The three vectors $[0,1]$, $[1,0]$ and $[1000,1000]$ are
   (A) Dependent,
   (B) Independent,
   (C) Pairwise orthogonal,
   (D) None of the above.

9. The function $f(.)$ is increasing over $[a,b]$. Then $[f(.)]^n$, where n is an odd integer greater than 1, is necessarily
   (A) Increasing over $[a,b]$,
   (B) Decreasing over $[a,b]$,
   (C) Increasing over $[a,b]$ if and only if $f(.)$ is positive over $[a,b]$,
   (D) None of the above.

10. The determinant of the matrix $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ is
   (A) 21,
   (B) -16,
   (C) 0,
   (D) 14.

11. In what ratio should a given line be divided into two parts, so that the area of the rectangle formed by the two parts as the sides is the maximum possible?
   (A) 1 is to 1,
   (B) 1 is to 4,
   (C) 3 is to 2,
   (D) None of the above.

12. Suppose (x^*, y^*) solves:

$$Minimize \;\; ax + by,$$

subject to

$$x^\alpha + y^\alpha = M,$$

and $x, y \geq 0$, where $a > b > 0$, $M > 0$ and $\alpha > 1$. Then, the solution is,

(A) $\frac{x*^{\alpha-1}}{y*^{\alpha-1}} = \frac{a}{b}$,
(B) $x^* = 0,\ y^* = M^{\frac{1}{\alpha}}$,
(C) $y^* = 0,\ x^* = M^{\frac{1}{\alpha}}$,
(D) None of the above.

13. Three boys and two girls are to be seated in a row for a photograph. It is desired that no two girls sit together. The number of ways in which they can be so arranged is
(A) $4_{P_2} \times 3!$,
(B) $3_{P_2} \times 2!$
(C) $2! \times 3!$
(D) None of the above.

14. The domain of x for which $\sqrt{x} + \sqrt{3-x} + \sqrt{x^2 - 4x}$ is real is,
(A) [0,3],
(B) (0,3),
(C) $\{0\}$,
(D) None of the above.

15. P (x) is a quadratic polynomial such that P (1) = P (-1). Then
(A) The two roots sum to zero,
(B) The two roots sum to 1,
(C) One root is twice the other,
(D) None of the above.

16. The expression $\sqrt{11+6\sqrt{2}} + \sqrt{11-6\sqrt{2}}$ is
(A) Positive and an even integer,
(B) Positive and an odd integer,
(C) Positive and irrational,
(D) None of the above.

17. What is the maximum value of $a(1-a)b(1-b)c(1-c)$, where a, b, c vary over all positive fractional values?

A 1,

B $\frac{1}{8}$,

C $\frac{1}{27}$,

D $\frac{1}{64}$.

18. There are four modes of transportation in Delhi: (A) Auto-rickshaw, (B) Bus, (C) Car, and (D) Delhi-Metro. The probability of using transports A, B, C, D by an individual is $\frac{1}{9}$, $\frac{2}{9}$, $\frac{4}{9}$, $\frac{2}{9}$ respectively. The probability that he arrives late at work if he uses transportation A, B, C, D is $\frac{5}{7}$, $\frac{4}{7}$, $\frac{6}{7}$, and $\frac{6}{7}$ respectively. What is the probability that he used transport A if he reached office on time?

    A $\frac{1}{9}$,

    B $\frac{1}{7}$,

    C $\frac{3}{7}$,

    D $\frac{2}{9}$.

19. What is the least (strictly) positive value of the expression $a^3+b^3+c^3-3abc$, where a, b, c vary over all strictly positive integers? (You may use the identity $a^3+b^3+c^3-3abc = \frac{1}{2}(a+b+c)\big((a-b)^2+(b-c)^2+(c-a)^2\big)$.)

    A 2,

    B 3,

    C 4,

    D 8.

20. If $a^2+b^2+c^2=1$, then $ab+bc+ca$ is,
    (A) -0.75,
    (B) Belongs to the interval $[-1,\ -0.5]$,
    (C) Belongs to the interval $[0.5,\ 1]$,
    (D) None of the above.

21. Consider the following linear programming problem:
    Maximize $a+b$ subject to
    $a+2b \leq 4$,
    $a+6b \leq 6$,

$a - 2b \leq 2$,
$a, b \geq 0$.
An optimal solution is:
(A) a=4, b=0,
(B) a=0, b=1,
(C) a=3,b=1/2,
(D) None of the above.

22. The value of $\int_{-4}^{-1} \frac{1}{x} dx$ equals,
(A) $ln\ 4$,
(B) Undefined,
(C) $ln(-4) - ln(-1)$,
(D) None of the above.

23. Given $x \geq y \geq z$, and $x + y + z = 9$, the maximum value of x + 3y + 5z is
(A) 27,
(B) 42,
(C) 21,
(D) 18.

24. A car with six sparkplugs is known to have two malfunctioning ones. If two plugs are pulled out at random, what is the probability of getting at least one malfunctioning plug.
(A) 1/15,
(B) 7/15,
(C) 8/15,
(D) 9/15.

25. Suppose there is a multiple choice test which has 20 questions. Each question has two possible responses - true or false. Moreover, only one of them is correct. Suppose a student answers each of them randomly.Which one of the following statements is correct?
(A) The probability of getting 15 correct answers is less than the probability of getting 5 correct answers,
(B) The probability of getting 15 correct answers is more than the

probability of getting 5 correct answers,
(C) The probability of getting 15 correct answers is equal to the probability of getting 5 correct answers,
(D) The answer depends on such things as the order of the questions.

26. From a group of 6 men and 5 women, how many different committees consisting of three men and two women can be formed when it is known that 2 of the men do not want to be on the committee together?
(A) 160,
(B) 80,
(C) 120,
(D) 200.

27. Consider any two consecutive integers a and b that are both greater than 1. The sum $(a^2 + b^2)$ is
(A) Always even,
(B) Always a prime number,
(C) Never a prime number,
(D) None of the above statements is correct.

28. The number of real non-negative roots of the equation

$$x^2 - 3|x| - 10 = 0$$

is,
(A) 2,
(B) 1,
(C) 0,
(D) 3.

29. Let $< a^n >$ and $< b^n >$, $n = 1, 2, \cdots$, be two different sequences, where $< a^n >$ is convergent and $< b^n >$ is divergent. Then the sequence $< a^n + b^n >$ is,
(A) Convergent,
(B) Divergent,
(C) Undefined,
(D) None of the above.

30. Consider the function

$$f(x) = \frac{|x|}{1+|x|}.$$

This function is,
(A) Increasing in x when $x \geq 0$,
(B) Decreasing in x,
(C) Increasing in x for all real x,
(D) None of the above.

Syllabus for ME II (Economics), 2012

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM Model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

Sample Questions for ME II (Economics), 2012

1. A price taking firm makes machine tools Y using labour and capital according to the following production function

$$Y = L^{0.25}K^{0.25}.$$

Labour can be hired at the beginning of every week, while capital can be hired only at the beginning of every month. It is given that the wage rate = rental rate of capital = 10. Show that the short run (week) cost function is $10Y^4/K^*$ where the amount of capital is fixed at K^* and the long run (month) cost function is $20Y^2$.

2. Consider the following IS-LM model
$C = 200 + 0.25Y_D$,
$I = 150 + 0.25Y - 1000i$,

$G = 250$,
$T = 200$,
$(m/p)^d = 2Y - 8000i$,
$(m/p) = 1600$,

where C = aggregate consumption, I = investment, G = government expenditures, T = taxes, $(m/p)^d$ = money demand, (m/p) = money supply, Y_D = disposable income $(Y - T)$. Solve for the equilibrium values of all variables. How is the solution altered when money supply is increased to $(m/p) = 1840$? Explain intuitively the effect of expansionary monetary policy on investment in the short run.

3. Suppose that a price-taking consumer A maximizes the utility function $U(x, y) = x^\alpha + y^\alpha$ with $\alpha > 0$ subject to a budget constraint. Assume prices of both goods, x and y, are equal. Derive the demand function for both goods. What would your answer be if the price of x is twice that of the price of y?

4. Assume the production function for the economy is given by

$$Y = L^{0.5} K^{0.5}$$

where Y denotes output, K denotes the capital stock and L denotes labour. The evolution of the capital stock is given by

$$K_{t+1} = (1 - \delta)K_t + I_t$$

where δ lies between 0 and 1 and is the rate of depreciation of capital. I represents investment, given by $I_t = sY_t$, where s is the savings rate. Derive the expression of steady state consumption and find out the savings rate that maximizes steady state consumption.

5. There are two goods x and y. Individual A has endowments of 25 units of good x and 15 units of good y. Individual B has endowments of 15 units of good x and 30 units of good y. The price of good y is Re. 1, no matter whether the individual buys or sells the good. The price of good x is Re. 1 if the individual wishes to sell it. It is, however, Rs. 3 if

the individual wishes to buy it. Let C_x and C_y denote the consumption of these goods. Suppose that individual B chooses to consume 20 units of good x and individual A does not buy or sell any of the goods and chooses to consume her endowment. Could A and B have the same preferences?

6. A monopolist has cost function $c(y) = y$ so that its marginal cost is constant at Re. 1 per unit. It faces the following demand curve

$$D(p) = \begin{cases} 0, \text{ if } & p > 20 \\ \dfrac{100}{p}, \text{ if } & p \leq 20. \end{cases}$$

   Find the profit maximizing level of output if the government imposes a per unit tax of Re. 1 per unit, and also the dead-weight loss from the tax.

7. A library has to be located on the interval $[0, 1]$. There are three consumers A, B and C located on the interval at locations 0.3, 0.4 and 0.6, respectively. If the library is located at x, then A, B and C's utilities are given by $-|x - 0.3|$, $-|x - 0.4|$ and $-|x - 0.6|$, respectively. Define a Pareto-optimal location and examine whether the locations $x = 0.1, x = 0.3$ and $x = 0.6$ are Pareto-optimal or not.

8. Consider an economy where the agents live for only two periods and where there is only one good. The life-time utility of an agent is given by $U = u(c) + \beta v(d)$, where u and v are the first and second period utilities, c and d are the first and second period consumptions and β is the discount factor. β lies between 0 and 1. Assume that both u and v are strictly increasing and concave functions. In the first period, income is w and in the second period, income is zero. The interest rate on savings carried from period 1 to period 2 is r. There is a government that taxes first period income. A proportion τ of income is taken away by the government as taxes. This is then returned in the second period to the agent as a lump sum transfer T. The government's budget is balanced i.e., $T = \tau w$. Set up the agent's optimization problem and write the first order condition assuming an interior solution. For given

values of r, β, w, show that increasing T will reduce consumer utility if the interest rate is strictly positive.

9. A monopolist sells two products, X and Y . There are three consumers with asymmetric preferences. Each consumer buys either one unit of a product or does not buy the product at all. The per-unit maximum willingness to pay of the consumers is given in the table below.

Consumer No.	X	Y
1	4	0
2	3	3
3	0	4.

The monopolist who wants to maximize total payoffs has three alternative marketing strategies: (i) sell each commodity separately and so charge a uniform unit price for each commodity separately (simple monopoly pricing);(ii) offer the two commodities for sale only in a package comprising of one unit of each, and hence charge a price for the whole bundle (pure bundling strategy), and (iii) offer each commodity separately as well as a package of both, that is, offer unit price for each commodity as well as charge a bundle price (mixed bundling strategy). However, the monopolist cannot price discriminate between the consumers. Given the above data, find out the monopolist's optimal strategy and the corresponding prices of the products.

10. Consider two consumers with identical income M and utility function $U = xy$ where x is the amount of restaurant good consumed and y is the amount of any other good consumed. The unit prices of the goods are given. The consumers have two alternative plans to meet the restaurant bill. Plan A: they eat together at the restaurant and each pays his own bill. Plan B: they eat together at the restaurant but each pays one-half of the total restaurant bill. Find equilibrium consumption under plan B.

11. Consider a community having a fixed stock X of an exhaustible resource (like oil) and choosing, over an infinite horizon, how much of this resource is to be used up each period. While doing so, the com-

munity maximizes an intertemporal utility function $U = \sum \delta^t ln(C_t)$ where C_t represents consumption or use of the resource at period t and $\delta (0 < \delta < 1)$ is the discount factor. Express the optimal consumption C_t for any period t in terms of the parameter δ and X.

12. A consumer, with a given money income M, consumes 2 goods x_1 and x_2 with given prices p_1 and p_2. Suppose that his utility function is $U(x_1, x_2) = Max(x_1, x_2)$. Find the Marshallian demand function for goods x_1, x_2 and draw it in a graph. Further, suppose that his utility function is $U(x_1, x_2) = Min(x_1, x_2)$. Find the income and the own price elasticities of demand for goods x_1 and x_2.

13. An economy, consisting of m individuals, is endowed with quantities $\omega_1, \omega_2, ..., \omega_n$ of n goods. The ith individual has a utility function $U(C_1^i, C_2^i, ...C_n^i) = C_1^i C_2^i ... C_n^i$, where C_j^i is consumption of good j of individual i.
(a) Define an allocation,a Pareto inferior allocation and a Pareto optimal allocation for this economy.
(b) Consider an allocation where $C_j^i = \lambda^i \omega_j$ for all j ,$\sum_i \lambda^i = 1$. Is this allocation Pareto optimal?

14. Suppose that a monopolist operates in a domestic market facing a demand curve $p = 5 - (\frac{3}{2})q_h$, where p is the domestic price and q_h is the quantity sold in the domestic market. This monopolist also has the option of selling the product in the foreign market at a constant price of 3. The monopolist has a cost function given by $C(q) = q^2$, where q is the total quantity that the monopolist produces. Suppose, that the monopolist is not allowed to sell more than 1/6 units of the good in the foreign market. Now find out the amount the monopolist sells in the domestic market and in the foreign market.

15. An economy produces two goods, food (F) and manufacturing (M). Food is produced by the production function $F = (L_F)^{1/2}(T)^{1/2}$, where L_F is the labour employed, T is the amount of land used and F is the amount of food produced. Manufacturing is produced by the production function $M = (L_M)^{1/2}(K)^{1/2}$, where L_M is the labour employed,

K is the amount of capital used and M is the amount of manufacturing production. Labour is perfectly mobile between the sectors (i.e. food and manufacturing production) and the total amount of labour in this economy is denoted by L. All the factors of production are fully employed. Land is owned by the landlords and capital is owned by the capitalists. You are also provided with the following data: $K = 36, T = 49$, and $L = 100$. Also assume that the price of food and that of manufacturing are the same and is equal to unity.
(a) Find the equilibrium levels of labour employment in the food sector and the manufacturing sector (i.e. L_F and L_M respectively)
(b)Next, we introduce a small change in the description of the economy given above. Assume that everything remains the same except for the fact that land is owned by none; land comes for free! How much labour would now be employed in the food and the manufacturing sectors?

16. Consider two countries - a domestic country (with excess capacity and unlimited supply of labour) and a benevolent foreign country. The domestic country produces a single good at a fixed price of Re.1 per unit and is in equilibrium initially (i.e., in year 0) with income at Rs. 100 and consumption, investment and savings at Rs. 50 each. Investment expenditure is autonomous. Final expenditure in any year t shows up as income in year t (say, Y_t) , but consumption expenditure in year t (say, C_t) is given by: $C_t = 0.5Y_{t-1}$.

    The foreign country agrees to give a loan of Rs.100 to the domestic country in year 1 at zero interest rate, but on conditions that it be (i) used for investment only and (ii) repaid in full at the beginning of the next year. The loan may be renewed every year, but on the same conditions as above. Find the income, consumption, investment and savings of the domestic country in year 1, year 2 and in final equilibrium when the country takes the loan in year 1 only.