SYLLABUS AND SAMPLE QUESTIONS FOR MSQE
(Program Code: MQEK and MQED)
2014

Syllabus for PEA (Mathematics), 2014

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations; (up to third degree).
Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.
Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.
Elementary Statistics: Elementary probability theory, measures of central tendency, dispersion, correlation and regression, probability distributions, standard distributions - Binomial and Normal.

Sample questions for PEA (Mathematics), 2014

1. Let $f(x) = \frac{1-x}{1+x}$, $x \neq -1$. Then $f(f(\frac{1}{x}))$, $x \neq 0$ and $x \neq -1$, is

   (a) 1,

   (b) x,

   (c) x^2,

   (d) $\frac{1}{x}$.

2. What is the value of the following definite integral?

$$2\int_0^{\frac{\pi}{2}} e^x \cos(x)dx.$$

   (a) $e^{\frac{\pi}{2}}$.

   (b) $e^{\frac{\pi}{2}} - 1$.

   (c) 0.

ECOPOINT.IN 9999886629, 9873903314

(d) $e^{\frac{\pi}{2}} + 1$.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as follows:

$$f(x) = |x - 1| + (x - 1).$$

Which of the following is not true for f?

(a) $f(x) = f(x')$ for all $x, x' < 1$.

(b) $f(x) = 2f(1)$ for all $x > 1$.

(c) f is not differentiable at 1.

(d) The derivative of f at $x = 2$ is 2.

4. Population of a city is 40 % male and 60 % female. Suppose also that 50 % of males and 30 % of females in the city smoke. The probability that a smoker in the city is male is closest to

(a) 0.5.

(b) 0.46.

(c) 0.53.

(d) 0.7.

5. A blue and a red die are thrown simultaneously. We define three events as follows:

- Event E: the sum of the numbers on the two dice is 7.
- Event F: the number on the blue die equals 4.
- Event G: the number on the red die equals 3.

Which of the following statements is true?

(a) E and F are disjoint events.

(b) E and F are independent events.

ECOPOINT.IN 9999886629, 9873903314

(c) E and F are not independent events.

(d) Probability of E is more than the probability of F.

6. Let $p > 2$ be a prime number. Consider the following set containing 2×2 matrices of integers:

$$T_p = \left\{ A = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} : a, b \in \{0, 1, \ldots, p-1\} \right\}.$$

A matrix $A \in T_p$ is p-special if determinant of A is not divisible by p. How many matrices in T_p are p-special?

(a) $(p-1)^2$.

(b) $2p - 1$.

(c) p^2.

(d) $p^2 - p + 1$.

7. A "good" word is any seven letter word consisting of letters from $\{A, B, C\}$ (some letters may be absent and some letter can be present more than once), with the restriction that A cannot be followed by B, B cannot be followed by C, and C cannot be followed by A. How many good words are there?

(a) 192.

(b) 128.

(c) 96.

(d) 64.

8. Let n be a positive integer and $0 < a < b < \infty$. The total number of real roots of the equation $(x-a)^{2n+1} + (x-b)^{2n+1} = 0$ is

(a) 1.

(b) 3.

ECOPOINT.IN
9999886629, 9873903314

(c) $2n - 1$.

(d) $2n + 1$.

9. Consider the optimization problem below:

$$\begin{aligned} \max_{x,y}\, & x + y \\ \text{subject to } & 2x + y \leq 14 \\ & -x + 2y \leq 8 \\ & 2x - y \leq 10 \\ & x, y \geq 0. \end{aligned}$$

The value of the objective function at optimal solution of this optimization problem:

(a) does not exist.

(b) is 8.

(c) is 10.

(d) is unbounded.

10. A random variable X is distributed in $[0, 1]$. Mr. Fox believes that X follows a distribution with cumulative density function (cdf) $F : [0, 1] \to [0, 1]$ and Mr. Goat believes that X follows a distribution with cdf $G : [0, 1] \to [0, 1]$. Assume F and G are differentiable, $F \neq G$ and $F(x) \leq G(x)$ for all $x \in [0, 1]$. Let $\mathbb{E}_F[X]$ and $\mathbb{E}_G[X]$ be the expected values of X for Mr. Fox and Mr. Goat respectively. Which of the following is true?

(a) $\mathbb{E}_F[X] \leq \mathbb{E}_G[X]$.

(b) $\mathbb{E}_F[X] \geq \mathbb{E}_G[X]$.

(c) $\mathbb{E}_F[X] = \mathbb{E}_G[X]$.

(d) None of the above.

ECOPOINT.IN 9999886629, 9873903314

11. Let $f : [0, 2] \to [0, 1]$ be a function defined as follows:

$$f(x) = \begin{cases} x & \text{if } x \leq \alpha \\ \frac{1}{2} & \text{if } x \in (\alpha, 2]. \end{cases}$$

where $\alpha \in (0, 2)$. Suppose X is a random variable distributed in $[0, 2]$ with probability density function f. What is the probability that the realized value of X is greater than 1?

(a) 1.

(b) 0.

(c) $\frac{1}{2}$.

(d) $\frac{3}{4}$.

12. The value of the expression

$$\sum_{k=1}^{100} \int_0^1 \frac{x^k}{k} dx$$

is

(a) $\frac{100}{101}$.

(b) $\frac{1}{99}$.

(c) 1.

(d) $\frac{99}{100}$.

13. Consider the following system of inequalities.

$$\begin{aligned} x_1 - x_2 &\leq 3 \\ x_2 - x_3 &\leq -2 \\ x_3 - x_4 &\leq 10 \\ x_4 - x_2 &\leq \alpha \\ x_4 - x_3 &\leq -4, \end{aligned}$$

where α is a real number. A value of α for which this system has a solution is

ECOPOINT.IN 9999886629, 9873903314

(a) -16.

(b) -12.

(c) -10.

(d) None of the above.

14. A fair coin is tossed infinite number of times. The probability that a head turns up for the first time after even number of tosses is

(a) $\frac{1}{3}$.

(b) $\frac{1}{2}$.

(c) $\frac{2}{3}$.

(d) $\frac{3}{4}$.

15. An entrance examination has 10 "true-false" questions. A student answers all the questions randomly and his probability of choosing the correct answer is 0.5. Each correct answer fetches a score of 1 to the student, while each incorrect answer fetches a score of zero. What is the probability that the student gets the mean score?

(a) $\frac{1}{4}$.

(b) $\frac{63}{256}$.

(c) $\frac{1}{2}$.

(d) $\frac{1}{8}$.

16. For any positive integer k, let S_k denote the sum of the infinite geometric progression whose first term is $\frac{(k-1)}{k!}$ and common ratio is $\frac{1}{k}$. The value of the expression $\sum_{k=1}^{\infty} S_k$ is

(a) e.

(b) $1+e$.

(c) $2+e$.

(d) e^2.

ECOPOINT.IN 9999886629, 9873903314

17. Let $G(x) = \int_0^x te^t dt$ for all non-negative real number x. What is the value of $\lim_{x\to 0} \frac{1}{x}G'(x)$, where $G'(x)$ is the derivative of G at x.

    (a) 0.
    (b) 1.
    (c) e.
    (d) None of the above.

18. Let $\alpha \in (0, 1)$ and $f(x) = x^\alpha + (1 - x)^\alpha$ for all $x \in [0, 1]$. Then the maximum value of f is

    (a) 1.
    (b) greater than 2.
    (c) in $(1, 2)$.
    (d) 2.

19. Let n be a positive integer. What is the value of the expression

$$\sum_{k=1}^{n} kC(n, k),$$

where $C(n, k)$ denotes the number of ways to choose k out of n objects?

    (a) $n2^{n-1}$.
    (b) $n2^{n-2}$.
    (c) 2^n.
    (d) $n2^n$.

20. The first term of an arithmetic progression is a and common difference is $d \in (0, 1)$. Suppose the k-th term of this arithmetic progression equals the sum of the infinite geometric progression whose first term is a and common ratio is d. If $a > 2$ is a prime number, then which of the following is a possible value of d?

ECOPOINT.IN 9999886629, 9873903314

(a) $\frac{1}{2}$.

(b) $\frac{1}{3}$.

(c) $\frac{1}{5}$.

(d) $\frac{1}{9}$.

21. In period 1, a chicken gives birth to 2 chickens (so, there are three chickens after period 1). In period 2, each chicken born in period 1 either gives birth to 2 chickens or does not give birth to any chicken. If a chicken does not give birth to any chicken in a period, it does not give birth in any other subsequent periods. Continuing in this manner, in period $(k+1)$, a chicken born in period k either gives birth to 2 chickens or does not give birth to any chicken. This process is repeated for T periods - assume no chicken dies. After T periods, there are in total 31 chickens. The maximum and the minimum possible values of T are respectively

(a) 12 and 4.

(b) 15 and 4.

(c) 15 and 5.

(d) 12 and 5.

22. Let a and p be positive integers. Consider the following matrix

$$A = \begin{bmatrix} p & 1 & 1 \\ 0 & p & a \\ 0 & a & 2 \end{bmatrix}$$

If determinant of A is 0, then a possible value of p is

(a) 1.

(b) 2.

(c) 4.

(d) None of the above.

ECOPOINT.IN
9999886629, 9873903314

23. For what value of α does the equation $(x-1)(x^2-7x+\alpha)=0$ have exactly two unique roots?

    (a) 6.
    (b) 10.
    (c) 12.
    (d) None of the above.

24. What is the value of the following infinite series?

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k^2} \log_e 3^k.$$

    (a) $\log_e 2$.
    (b) $\log_e 2 \log_e 3$.
    (c) $\log_e 6$.
    (d) $\log_e 5$.

25. There are 20 persons at a party. Each person shakes hands with some of the persons at the party. Let K be the number of persons who shook hands with odd number of persons. What is a possible value of K?

    (a) 19.
    (b) 1.
    (c) 20.
    (d) All of the above.

26. Two independent random variables X and Y are uniformly distributed in the interval $[0,1]$. For a $z \in [0,1]$, we are told that probability that $\max(X,Y) \leq z$ is equal to the probability that $\min(X,Y) \leq (1-z)$. What is the value of z?

    (a) $\frac{1}{2}$.

ECOPOINT.IN 9999886629, 9873903314

(b) $\frac{1}{\sqrt{2}}$.

(c) any value in $[\frac{1}{2}, 1]$.

(d) None of the above.

27. Let $f : \mathbb{R} \to \mathbb{R}$ be a function that satisfies for all $x, y \in \mathbb{R}$

$$f(x+y)f(x-y) = \big(f(x) + f(y)\big)^2 - 4x^2 f(y).$$

Which of the following is not possible for f?

(a) $f(0) = 0$.

(b) $f(3) = 9$.

(c) $f(5) = 0$.

(d) $f(2) = 2$.

28. Consider the following function $f : \mathbb{R} \to \mathbb{Z}$, where $\mathbb{R}$ is the set of all real numbers and $\mathbb{Z}$ is the set of all integers.

$$f(x) = \lceil x \rceil,$$

where $\lceil x \rceil$ is the smallest integer that is larger than x. Now, define a new function g as follows. For any $x \in \mathbb{R}$, $g(x) = |f(x)| - f(|x|)$, where $|x|$ gives the absolute value of x. What is the range of g?

(a) $\{0, 1\}$.

(b) $[-1, 1]$.

(c) $\{-1, 0, 1\}$.

(d) $\{-1, 0\}$.

29. The value of $\lim_{x \to -1} \frac{x+1}{|x+1|}$ is.

(a) 1.

(b) -1.

(c) 0.

ECOPOINT.IN
9999886629, 9873903314

(d) None of the above.

30. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = 2$ if $x \leq 2$ and $f(x) = a^2 - 3a$ if $x > 2$, where a is a positive integer. Which of the following is true?

    (a) f is continuous everywhere for some value of a.

    (b) f is not continuous.

    (c) f is differentiable at $x = 2$.

    (d) None of the above.

ECOPOINT.IN 9999886629, 9873903314

Syllabus for PEB (Economics), 2014

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM Model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

Sample questions for PEB (Economics), 2014

1. Consider a firm that can sell in the domestic market where it is a monopolist, and/or in the export market. The domestic demand is given by $p = 10 - q$, and export price is 5. Suppose the firm has a constant marginal cost of 4 and a capacity constraint on output of 100.

   (a) Solve for the optimal production plan of the firm. [**15 marks**]

   (b) Solve for the optimal production plan of the firm if its constant marginal cost is 6. [**5 marks**]

2. (a) Consider a consumer who can consume either A or B, with the quantities being denoted by a and b respectively. If the utility function of the consumer is given by

   $$-\left[(10-a)^2 + (10-b)^2\right].$$

   Suppose prices of both the goods are equal to 1.

   i. Solve for the optimal consumption of the consumer when his income is 40. [**10 marks**]

   ii. What happens to his optimal consumption when his income goes down to 10. [**5 marks**]

   (b) A monopolist faces the demand curve $q = 60 - p$ where p is measured in rupees per unit and q in thousands of units. The monopolist's total cost of production is given by $C = \frac{1}{2}q^2$.

ECOPOINT.IN
9999886629, 9873903314

i. What is the deadweight loss due to monopoly? [**3 marks**]

ii. Suppose a government could set a price ceiling (maximum price) that the monopolist can charge. Find the price ceiling that the government should set to minimize the deadweight loss. [**2 marks**]

3. (a) A cinema hall has a capacity of 150 seats. The owner can offer students a discount on the price when they show their student IDs. The demand for tickets from students is

$$D_s = 220 - 40P_s,$$

where P_s is the price of tickets for students after the discount. The demand for tickets for non-students is

$$D_n = 140 - 20P_n,$$

where P_n is the price of tickets for non-students.

i. What is the maximum profit the owner can make? [**8 marks**]

ii. What is the maximum profit he could make if the demand functions of students and non-students were interchanged? [**4 marks**]

(b) There are 11 traders and 6 identical (indivisible) chickens. Each trader wants to consume at most one chicken. There is also a (divisible) good called "money". Let D_i equal to 1 indicate that trader i consumes a chicken; 0 if he does not. Trader $i's$ utility function is given by $u_iD_i + m_i$, where u_i is the value he attaches to consuming a chicken, m_i is the units of money that the trader has. The valuations for the 11 traders are:

$u_1 = 10$; $u_2 = 8$; $u_3 = 7$; $u_4 = 4$; $u_5 = 3$; $u_6 = 1$; $u_7 = u_8 = 3$; $u_9 = 5$; $u_{10} = 6$; $u_{11} = 8$.

Initially each trader is endowed with 25 units of money. Traders $6, 7, 8, 9, 10, 11$ are endowed with one chicken each.

i. What is a possible equilibirum market price (units of money per chicken) in a competitive market? [**4 marks**]

ECOPOINT.IN 9999886629, 9873903314

ii. Is the equilibrium unique? [**4 marks**]

4. (a) Consider a monopolist who faces a market demand for his product:

$$p(q) = 20 - q,$$

where p is the price and q is the quantity. He has a production function given by

$$q = \min\left\{\frac{L}{2}, \frac{K}{3}\right\},$$

where L denotes labour and K denotes capital. There is a physical restriction on the availability of capital, that is, $\overline{K}$. Let both wage rates (w) and rental rates (r) be equal to 1. Find the monopoloy equilibrium quantity and price when (i) when $\overline{K} = 24$; (ii) $\overline{K} = 18$. [**12 marks**]

(b) Define Samuelson's Weak Axiom of Revealed Preference (WARP). [**2 marks**]

(c) Prove that WARP implies non-positivity of the own-price substitution effect and the demand theorem. [**6 marks**]

5. Consider two firms: 1 and 2, with their output levels denoted by q_1 and q_2. Suppose both have identical and linear cost functions, $C(q_i) = q_i$. Let the market demand function be $q = 10 - p$, where q denotes aggregate output and p the market price.

(a) Suppose the firms simultaneously decide on their output levels. Define the equilibrium in this market. Solve for the reaction functions of the two fims. Using these, find the equilibrium. [**10 marks**]

(b) Suppose the firms still compete over quantities, but both have a capacity constraint at an output level of 2. Find these reaction functions and the equilibrium in this case. [**10 marks**]

6. (a) Suppose the government subsidizes housing expenditures of low-income families by providing them a rupee-for-rupee subsidy for

ECOPOINT.IN 9999886629, 9873903314

their expenditure. The Lal family qualifies for this subsidy. They spend Rs. 250 on housing, and receive Rs. 250 as subsidy from the government.

Recently, a new policy has been proposed to replace the earlier policy. The new policy proposes to provide each low income family with a lump-sum transfer of Rs. 250, which can be used for housing or other goods.

i. Explain graphically if the Lal family would prefer the current program over the proposed program. [**6 marks**]

ii. Can they be indifferent between the two programs? [**3 marks**]

iii. Does the optimal consumption of housing and other goods change compared to the subsidy scheme? If yes, how? [**3 marks**]

(b) A drug company company is a monopoly supplier of Drug X which is protected by a patent. The demand for the drug is

$$p = 100 - X$$

and the monopolist's cost function is

$$C = 25 + X^2$$

i. Determine the profit maximizing price and quantity of the monopolist. [**2 marks**]

ii. Suppose the patent expires at a certain point in time, and after that any new drug company can enter the market and produce Drug X, facing the same cost function. What will be the competitive equilibrium industry output and price? How many firms will be there in the market? [**6 marks**]

7. Assume that an economy's production function is given by

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}$$

where Y_t is output at time t, K_t is the capital stock at time t and N is the *fixed* level of employment (number of workers), $\alpha \in (0, 1)$ is the

ECOPOINT.IN 9999886629, 9873903314

share of output paid to capital. The evolution of the capital stock is given by

$$K_{t+1} = (1-\delta) K_t + I_t$$

where I_t is investment at time t and $\delta \in [0,1]$ is the depreciation rate.

(a) Derive an expression for $\frac{Y}{N}$. [**5 marks**]

(b) How large is the effect of an increase in the savings rate on the steady state level of output per worker. [**10 marks**]

(c) What is the savings rate that would maximize steady state consumption per worker? [**5 marks**]

8. In an IS-LM model, graphically compare the effect of an expansionary monetary policy with an expansionary fiscal policy on investment (I) in (1) the short-run and (2) the medium run (where the aggregate supply and aggregate demand curves adjust). Assume that

$$I = I(i, Y),$$

where i is the interest rate and Y is the output. Also, $\frac{\partial I}{\partial i} < 0$ and $\frac{\partial I}{\partial Y} > 0$. [**15 marks**]

Under which policy (expansionary monetary or fiscal), is the investment higher in the medium run? [**5 marks**]

9. Suppose the economy is characterized by the following equations:

$$C = c_0 + c_1 Y_D$$
$$Y_D = Y - T$$
$$I = b_0 + b_1 Y,$$

where C is consumption, Y is the income, Y_D is the disposable income, T is tax, I is investment, and c_0, c_1, b_0, b_1 are positive constants with $c_1 < 1, b_1 < 1$. Government spending is constant.

(a) Solve for equilibrium output. [**5 marks**]

ECOPOINT.IN 9999886629, 9873903314

(b) What is the value of the multiplier? For the multiplier to be positive, what condition must $c_1 + b_1$ satisfy? [**5 marks**]

(c) How will equilibrium output be affected when b_0 is changed? What will happen to saving? [**5 marks**]

(d) Instead of fixed T, suppose $T = t_0 + t_1 Y$, where $t_0 > 0$ and $t_1 \in (0, 1)$. What is the effect of increase in b_0 on equilibrium Y? Is it larger or smaller than the case where taxes are autonomous? [**5 marks**]

10. Consider an economy where a representitive agent lives for three periods. In the first period, she is young - this is the time when she gets education. In the second period, she is middle-aged and with the level of education acquired in the first period, she generates income. More specifically, if she has h units of education in the first period, she can earn $\overline{w}h$, in the second period, where $\overline{w}$ is the exogenously given wage rate.

    The agent borrows funds for her education when she is young and repays with interest when she is middle aged. If in the first period, the agent borrows e, then the human capital h at the beginning of the second period becomes $h(e)$, where $\frac{dh}{de} > 0$ along with $\frac{d^2h}{de^2} < 0$.

    In the third period of her life, she consumes out of her savings made in the second period, that is, when she was middle aged. Assume that the exogenous rate of interest (gross) on saving or borrowing is $\overline{R}$. For simplicity, assume that an agent does not consume when she is young and, thus, the life time utility is $u(c^M) + \beta u(c^O)$, where c^M and c^O are the level of consumption when they are middle-aged and old respectively and $\beta \in (0, 1)$ is the discount factor.

    (a) Write down the utility maximization problem of the agent and the first order conditions. [**10 marks**]

    (b) How does the optimal level of education vary with the wage rate and the rate of interest? [**10 marks**]

ECOPOINT.IN
9998866629, 9873903314