2017

SAMPLE QUESTIONS: PEA (MATHEMATICS)

- For each of the 30 questions, there are four suggested answers. Only one of the suggested answers is correct. You will have to identify the correct answer to get full credit for that question. Indicate your choice of the correct answer by darkening the appropriate oval completely on the answer-sheet.
- You will get:

  4 marks for each **correctly** answered question,

  0 marks for each **incorrectly** answered question, and

  1 mark for each **unanswered** question.

1. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are two functions such that $f(x) = ax+b$ and $g(x) = cx+d$, then $f(g(x)) = g(f(x))$ holds if and only if

   A. $f(a) = g(c)$ B. $f(b) = g(b)$ C. $f(d) = g(b)$ D. $f(c) = g(a)$

2. A box contains 90 good and 10 defective screws. If 10 screws are drawn without replacement, the probability that none of them is defective is

   A. $\frac{^{90}C_{10}}{^{100}C_{10}}$ B. $\frac{90^{10}}{100^{10}}$ C. $\frac{^{90}P_{10}}{^{100}P_{10}}$ D. None of the above

3. The number of ordered pairs of integers (x, y) satisfying the equation

   $$x^2 + 6x + y^2 = 4$$

   is

   A. 2 B. 4 C. 6 D. 8

4. The value of $\lim_{x\to-\infty} \frac{3x^2-Sin(5x)}{x^2+2}$ is

   A. 0 B. 1 C. 2 D. 3

5. The smallest integer that produces remainders of $2, 4, 6$ and 1 when divided by $3, 5, 7$ and 11 respectively is

   A. 104 B. 1154 C. 419 D. None of the above

6. Given thirty people, the probability that among the twelve months there are six containing two birthdays each and six containing three each is

   A. $\frac{30!}{2^6 6^6} \times^{12} C_6 \times 12^{-30}$ B. $^{12}C_6 \times^{30} C_{12}$ C. $\frac{30!}{2^6 6^6} \times^{12} C_6 \times \frac{1}{^{30}C_{12}}$ D. None of the above

7. Let $[x]$ denote the greatest integer less than or equal to x for any real number x. Then the number of solutions of $|x^2 - [x]| = 1$ is

   A. 0 B. 1 C. 2 D. 3

8. Let $a, b, c \in \mathbb{R}$, $a^2 + b^2 + c^2 = 1$, and $A = ab + bc + ca$. Then

   A. $-\frac{1}{2} < A < 1$ B. $-1 < A < 1$ C. $-\frac{1}{2} < A \leq 1$ D. $-\frac{1}{2} \leq A \leq 1$

9. Two dice are rolled. If the two faces are different, the probability that one is a six is

   A. $\frac{5}{6}$ B. $\frac{2}{3}$ C. $\frac{1}{2}$ D. $\frac{1}{3}$

10. The minimum value of $\frac{x^2+2}{\sqrt{x^2+1}}$ (where x is a real number) is

    A. 1 B. 2 C. $\sqrt{2}$ D. None of the above

11. If $f(x) = (\frac{a+x}{b+x})^{a+b+2x}$, then $f'(0)$ equals

    A. $(2\log(\frac{a}{b}) + \frac{b^2-a^2}{ab}) \times (\frac{a}{a+b})^{a+b}$ B. $(2\log(\frac{a}{b}) + \frac{b^2-a^2}{ab}) \times (\frac{a}{b})^{a+b}$ C. $(2\log(\frac{a}{a+b}) + \frac{b^2-a^2}{ab}) \times (\frac{a}{b})^{a+b}$ D. None of the above

12. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random without replacement. Then the probability that none of the balls drawn is blue is

    A. 10/21 B. 11/21 C. 2/7 D. 5/7

13. The letters of the word COCHIN are permuted and all the permutations are arranged lexicographically (i.e., in alphabetical order as in an English dictionary). The number of words that appear before the word COCHIN is

    A. 96 B. 360 C. 192 D. 48

14. The function $f : [0, \infty) \times [0, \infty) \to \mathbb{R}$ given by $f(x, y) = x^{1/3}y^{1/2}$ is

    A. convex and quasi-convex B. convex but not quasi-convex C. quasi-convex but not convex D. neither quasi-convex nor convex

15. The function in the previous question is

    A. homogeneous of degree 1 and homothetic B. homogeneous of degree 1 but not homothetic C. homothetic but not homogeneous of degree 1 D. Neither homogeneous of degree 1 nor homothetic

16. Player 1 and Player 2 both start with 100 rupees. Each round of a game consists of the following:
    Both players choose a number randomly and independently from 1 to 5. If both players choose the same number, then Player 1 gives rupees 10 to Player 2. Otherwise, Player 2 gives rupees 10 to Player 1. Then the expected amount of money Player 1 will be left with after playing 10 rounds of this game is

    A. 120 B. 100 C. 50 D. 160

17. Let $f : (0, 1) \to \mathbb{R}$ be a function defined by

$$f(x) = \left\{ \begin{array}{ll} x^2, & \text{if } x \text{ is rational;} \\ 2 - x^2, & \text{otherwise.} \end{array} \right\}$$

    Then f is continuous at

    A. no point in (0, 1) B. exactly one point in (0, 1) C. exactly two points in (0, 1) D. more than two points in (0, 1)

18. The last digit of 432^{43567} is

A. 2 B. 4 C. 6 D. 8

19. Two squares are chosen at random on a chessboard (with 64 squares). Then the probability that they have a side in common is

A. 1/9 B. 1/27 C. 1/18 D. None of the above

20. Let $f : [0, 1] \to \mathbb{R}$ be a function such that $f(x) = \frac{x}{x-2}$. Then f is

A. one-one B. onto C. one-one and onto D. None of the above

21. Consider the functions $f : \mathbb{R}^2 \to \mathbb{R}^3$ and $g : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x, y) = (x + 2y, x - y, -2x + 3y)$ and $g(x, y) = (x + 1, y + 2)$. A. Both f and g are linear transformations B. f is a linear transformation, but g is not a linear transformation C. f is not a linear transformation, but g is a linear transformation D. Neither f nor g are linear transformations

22. The ratio of boys to girls at birth in Singapore is 1.09:1 Then the proportion of Singaporean families with exactly 6 children who will have at least 3 boys is

A. 0.696 B. 0.315 C. 0.521 D. 0.455

23. Let

$$X = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}.$$

Then

A.
$$X^{-1} = 1/8 \begin{pmatrix} 4 & -2 & 0 \\ 0 & 4 & -6 \\ 0 & 0 & 4 \end{pmatrix}$$

B.
$$X^{-1} = 1/8 \begin{pmatrix} 4 & 0 & 0 \\ -2 & 4 & 0 \\ 3 & -6 & 4 \end{pmatrix}$$

C.
$$X^{-1} = 1/8 \begin{pmatrix} 4 & -2 & 3 \\ 0 & 4 & -6 \\ 0 & 0 & 4 \end{pmatrix}$$

D. X^{-1} does not exist

24. Suppose the probability of having a girl is 1/2 and so is the probability of having a boy. Now consider a family with two children. Then the probability that both the children are girls given that at least one of them is a girl is

A. 1/4 B. 2/3 C. 1/3 D. 1/2

25. Let $U : \mathbb{R}_+ \to \mathbb{R}$ be a strictly increasing function such that $U(x) \neq -1$ for all $x \in \mathbb{R}_+$, where $\mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$. Then the function $V : \mathbb{R}_+ \to \mathbb{R}$, defined by $V(x) = \frac{U(x)}{1+U(x)}$, is

A. necessarily strictly increasing B. necessarily strictly decreasing C. necessarily constant D. None of the above

26. A box contains three coins: two regular coins and one fake two-headed coin (i.e., $P(H) = 1$). Bagha picks a coin at random and tosses it, and gets head. Then the probability that it is the two-headed coin is

A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{1}{2}$ D. None of the above

27. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(x)f'(x) < 0$ for all $x \in \mathbb{R}$. Then A. $f(x)$ is an increasing function B. $|f(x)|$ is an increasing function C. $f(x)$ is a decreasing function D. $|f(x)|$ is a decreasing function

28. Let X and Y be two independent discrete random variables with the CDFs F_X and F_Y. Then the CDF of $W = \min\{X, Y\}$ is

A. $F_W(w) = \frac{F_X(w)+F_Y(w)}{2}$ B. $F_W(w) = \min\{F_X(w), F_Y(w)\}$ C. $F_W(w) = F_X(w)F_Y(w)$ D. None of the above

29. Two dice are thrown simultaneously. Then the probability of getting two numbers whose product is even is

A. 1/8 B. 1/4 C. 3/4 D. None of the above

30. Let $[x]$ denote the greatest integer less than or equal to x for any real number x. The range of the function $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = \frac{\sin(\pi[x])}{x^2+5}$, is

A. $(-1, 1)$ B. $[-1, 1]$ C. $\{-1, 1\}$ D. $\{0\}$

PEB (ECONOMICS) --- SAMPLE QUESTIONS 2017

For each of the thirty questions, there are four possible answers. You will get 4 marks for each correctly answered question, 1 mark for each unanswered question, and 0 marks for each incorrectly answered question.

Question 1: Under nominal wage rigidity, the short run aggregate supply schedule will be

(a) Vertical
(b) Horizontal
(c) Upward sloping
(d) Downward sloping

Question 2: Under rational expectations and no nominal rigidities, aggregate output is sensitive to _____ supply and _____ demand shocks.

(a) Anticipated, anticipated
(b) Anticipated, unanticipated
(c) Unanticipated, anticipated
(d) Unanticipated, unanticipated

Question 3: Keynes argued that monetary policy was ineffective during the Great Depression because

(a) IS curve was vertical and stuck at a low level of income.
(b) Both the IS and the LM curves were vertical.
(c) IS curve was continuously shifting, while the LM curve was vertical.
(d) None of the above choices is correct.

Question 4: In the basic Solow model of growth

(a) An increase in the savings rate raises the steady-state growth rate
(b) An increase in the growth rate of population lowers the steady-state growth rate
(c) An increase in the growth rate of population has no impact on the steady-state growth rate
(d) An increase in the savings rate has no impact on the steady-state growth rate.

Question 5: The money multiplier is _____ in the reserve-deposit ratio and _____in the cash-deposit ratio.

(a) Increasing; Decreasing
(b) Decreasing; Decreasing
(c) Decreasing; Increasing
(d) Increasing; Increasing

Question 6: If the IS curve is downward sloping and the LM curve is vertical, a unit increase in government expenditure results in

(a) Crowding in and higher increase in equilibrium income
(b) No crowding out and equivalent increase in the equilibrium income

(c) Partial crowding out and lower increase in equilibrium income
(d) Complete crowding out and no increase in equilibrium income

Question 7: Monetary policy is completely ineffective in raising output if

(a) The IS curve is horizontal and the LM curve is upward sloping
(b) The IS curve is vertical and the LM curve is upward sloping
(c) The IS curve is downward sloping and the LM curve is upward sloping
(d) The IS curve is downward sloping and the LM curve is vertical.

Question 8: Which of the following statements is correct? In a closed economy, fiscal policy is more effective

(a) The smaller the induced change in interest rates and smaller the responsiveness of investment to these changes.
(b) The larger the induced change in interest rates and smaller the responsiveness of investment to these changes.
(c) The smaller the induced change in interest rates and larger the responsiveness of investment to these changes.
(d) The larger the induced change in interest rates and lager the responsiveness of investment to these changes.

Question 9: Which of the following spells the most fundamental difference between the standard Solow model of growth and the standard optimal growth model?

(a) The rate of technology progress is endogenous in the former but exogenous in the latter
(b) The savings rate is exogenous in the former, but endogenous in the latter
(c) Capital utilization is exogenous in the former, but endogenous in the latter
(d) All of the above.

Question 10: Consider a simple Keynesian medel where equilibrium output is determined by aggregate demand. Investment is autonomous and a constant proportion of the income is saved. In this framework an increase in the savings propensity has the following effect:

(a) It leads to higher level of output in the new equilibrium
(b) It leads to lower level of output in the new equilibrium
(c) The level of output in the new equilibrium remains unchanged
(d) The level of output in the new equilibrium may increase or decrease depending on the degree of increase in the savings propensity.

The following pertains to Questions 11-14: Consider an agrarian economy consisting of two single membered households. The households are engaged in own cultivation using their family land, labour and capital. Each household is endowed with 1 acre of land and 1unit of labour. However the two households differ in terms of their initial capital endowments (K^R_0 and K_0^P), where R denotes the relatively richer household and P denotes the poorer household. Assume that $2<K^R_0<4$, and

$0<K_0^P<1$. The households have access to two technologies, which are specified by the following production functions:

Technology A: $Y_t=(N_tL_t)^{1/2}(K_t)^2$;

Technology B: $Y_t=(N_tL_t)^{1/2}(K_t)^{1/2}$.

Where N_t represents land (in acres), L_t represents labour, and K_t represents capital in period t.

The households choose the technology that gives them higher output(given their land, labour and capital stock) in any period t. In every period they consume half of their total income and save and invest the rest, which adds to the next period's capital stock. Land and labour stock remain constant over time. Existing capital stock depreciates fully upon production.

Question 11: Given their initial factor endowments, the technology choices of the rich household and poor household respectively are as follows:

(A) Household R chooses technology A; household P chooses B
(B) Household R chooses technology B; household P chooses A
(C) Both households choose technology A
(D) Both households choose technology B

Question 12: In the short run, the average capital stock in the economy (K) evolves according to the following dynamic path:

(A) $dK/dt=1/4[(K^R_t)^{1/2}+(K^p_t)^2-2(K^R_t+K^p_t)]$
(B) $dK/dt=1/4[(K^R_t)^2+(K^p_t)^2-2(K^R_t+K^p_t)]$
(C) $dK/dt=1/4[(K^R_t)^{1/2}+(K^p_t)^{1/2}-2(K^R_t+K^p_t)]$
(D) $dK/dt=1/4[(K^R_t)^2+(K^p_t)^{1/2}-2(K^R_t+K^p_t)]$

Question 13: In the long run:

(A) Income of both households grows perpetually
(B) Income of household R grows perpetually while income of household P approaches a constant.
(C) Income of household P grows perpetually while income of household R approaches a constant.
(D) Income of household R grows perpetually while income of household P falls perpetually.

Question 14: If at the end of the initial time period, the households were given a choice to spend their savings in buying more land instead of investing in capital stock:

(A) Both households would have bought more land
(B) Both households would have still invested in capital
(C) Household R would have still invested in capital but household P would have bought more land

(D) Household P would have still invested in capital but household R would have bought more land

Question 15: In the Mundell-Fleming model of a small open economy with flexible exchange rates and perfect capital mobility, suppose the economy is initially in equilibrium. If lump sum taxes are increased, what happens to the equilibrium levels of the country's (i) GDP (ii) interest rate and (iii) exchange rate?

(a) (i) falls, (ii) falls,(iii) appreciates
(b) (i) and (ii) remain unchanged; (iii) depreciates
(c) (i) falls,(ii) and (iii) remain unchanged
(d) All three remain unchanged.

Question 16: Raju consumes goods 1 and 2. Raju thinks that 2 units of good 1 is always a perfect substitute for 3 units of good 2. Which of the following utility functions is the only one that would NOT represent Raju's preferences?

(a) $U(x_1,x_2)= 9x_1^2+12x_1 x_2+4x_2^2$.
(b) $U(x_1,x_2)= \min\{3x_1, 2x_2\}$
(c) $U(x_1,x_2)= 30x_1+20x_2-10,000$.
(d) More than one of the above does NOT represent Raju's preferences.

Question 17: Riya has a demand function for mango juice given by q= .02m-2p, where m is income and p is price. Riya's income is 6,000 and she initially had to pay a price of 30 per bottle of mango juice. The price of mango juice rose to 40. The substitution effect of the price change

(a) Reduced her demand by 20.
(b) Increasedher demand by 20.
(c) Reduced her demand by 8.
(d) Reduced her demand by 32.

Question18: Ankita has a utility function $U(c_1,c_2)=c_1^{1/2}+ 0.83c_2^{1/2}$, where c_1 is her consumption in period 1 and c_2 is her consumption in period 2. Her income in period 1 is 2 times as large as her income in period 2. At what interest rate will she choose to consume the same amount in period 1 as in period 2?

(a) 0.40
(b) 0.10
(c) 0.20
(d) 0

Question 19: In a crowded city long ago, the civic authorities decided that rents were too high. The long run supply function of two-room rental apartments was given by q=18+2p and the long-run demand fuction was given by q=114-4p where p was the rental rate in rupees per week. The authorities made it illegal torent an apartment for more than 10 rupees per week. To avoid a housing shortage, the authorities agreed to pay landlords enough of a subsidy to make supply equal to demand. How much would the weekly subsidy per apartment have to be to eliminate excess demand at the ceiling price?

(a) 9
(b) 15
(c) 18
(d) 36

Question 20: A firm has the production function $f(x,y)=x^{0.70}y^{-0.30}$. This firm has

(a) Decreasing returns to scale and diminishing marginal product for factor x.
(b) Increasing returns to scale and decreasing marginal product of factor x.
(c) Constant returns to scale.
(d) None of the other options are correct.

Question 21: The production function is $f(x_1,x_2)=x_1^{1/2}x_2^{1/2}$. If the price of factor 1 is 8 and the price of factor 2 is 16, in what proportions should the firm use factors 1 and 2 if it wants to maximize profits?

(a) $x_1=x_2$.
(b) $x_1= 0.50\ x_2$
(c) $x_1=2x_2$.
(d) We can't tell without knowing the price of output.

Question 22: The supply curve of any firm *i* in a competitive industry is $S_i(p) =p/2$. If a firm produces 3 units of output, what are its total variable costs?

(a) 18
(b) 7
(c) 9
(d) There is not enough information given to determine total variable costs.

Question 23: The demand for slops is given by the equation q=14-p. Slops can be made at zero marginal cost. But before any slops can be produced, the firm must undertake a fixed cost of 54. Since the inventor has a patent on slops, it can be a monopolist in this new industry.

(a) The firm will produce 7 units of Slops
(b) A Pareto improvement could be achieved by having the government pay the firm a subsidy of 59 and insisting that the firm offer slops at zero price.
(c) From the point of view of social efficiency, it is best that no slops be produced.
(d) None of the other options are correct.

Question 24: A price-discriminating monopolist sells in two separate markets such that goods sold in one market are never resold in the other. It charges a price of 4 in one market and a price of 8 in the other market. At these prices, the price elasticity in the first market is -1.50 and price elasticity in the second market is -0.10. Which of the following actions is sure to raise the monopolists profits?

(a) Raise p_2
(b) Raise p_1 and Lower p_2
(c) Raise both p_1 and p_2
(d) Raise p_2 and lower p_1.

Question 25: Suppose that A and B go into the wine business in a small country where wine is difficult to grow. The demand for wine is given by p=Rs 360-.02Q where p is the price and Q is the total quantity sold. The industry consists of just the two Cournot duopolists, A and B. Imports are prohibited. A has constant marginal costs of Rs 15 and B has marginal costs of Rs. 75. How much is A's output in equilibrium?

(a) 675
(b) 1,350
(c) 337.50
(d) 1012.50

Question 26: On a certain island there are only two goods, wheat and milk.The only scarce resource is land. There are 1,000 acres of land. An acre of land will produce either 16 units of milk or 37 units of wheat. Some citizens have lots of land, some have just a little bit. The citizens of the island all have utility functions of the form U(M,W)=MW. At every pareto-optimal allocation,

(a) The number of units of milk produced equals the number of units of wheat produced.
(b) Total milk production is 8,000
(c) Every consumer's marginal rate of substitution between milk and wheat -1.
(d) None of the above is true at every pareto optimal allocation.

Question 27: Kabir's utility is $U(c,d,h) = 2c+5d-d^2-2h$, where d is the number of hours per day that he spends driving around, h is the number of hours per day spent driving around by the other people in his home town and c is the amount of money he has left to spend on other stuff besides petrol and auto repairs. Petrol and auto repairs cost Rs.50 per hour of driving . All the people in Kabir's home town have the same tastes. If each citizen believes that his own driving will not affect the amount of driving done by the others, they will all drive D1 hours per day. If they all drive the same amount, they would all be best off if each drove D2 hours per day, where

(a) D1=2 and D2=1
(b) D1=D2=2
(c) D1=4 and D2=2
(d) D1=5 and D2=0

Question 28: An airport is located next to a housing development. Where X is the number of planes that land per day and Y is the number of houses in the housing development, profits of the airport are $22X-X^2$ and profits of the developer are $32Y-Y^2-XY$. Let H1 be the number of houses built if a single profit-maximizing company owns the airport and the housing development. Let H2 be the number of houses built if the airport has to pay the developer the total "damages"XY done by the planes to the developer's profits. Then

(a) H1=H2=14
(b) H1=14 and H2=16
(c) H1=16 and H2=14
(d) H1=16 and H2=15

Question 29: A town with a population of 500 has a single public good and a single private good. Everyone's utility function is Ui(Xi,Y)=Xi-64/Y, where Xi is the amount of private good consumed by i and Y is the amount of the public good. The price of the private good is Re 1 per unit. The cost of the public good to the city is Rs. 5 per unit. Everyone has an income of at least Rs. 5,000. What is the Pareto-efficient amount of the public good for the town to provide?

(a) 80 square meters
(b) 200 square meters
(c) 100 square meters
(d) None of the other options are correct.

Question 30: Akash has the utility function U(b,w)= 6b+24w and Akshey has the utility function U(b,w)=bw. If we draw an Edgeworth Box with b on the horizontal axis and w on the vertical axis and if we measure Akash's consumptions from the lower left corner of the box, then the contract curve contains

(a) A straight line running from the upper right corner of the box to the lower left.
(b) A curve that gets steeper as you move from left to right.
(c) A straight line with slope ¼ passing through the upper right corner of the box.
(d) A curve that gets flatter as you move from left to right.