1. Consider $f : \Re \to \Re$ defined by $f(x) = \sum_{i=1}^{11} |x - a_i|$ for all $x \in \Re$, where $a_1 < .... < a_{11}$. This function has a minimum when x equals :

(A) a_2 (B) a_6

(C) a_7 (D) a_{11}

2. Consider the set $\{(x, y) \in \Re_+ \times \Re \mid y \leq \ln x - e^x\}$. This set is :

(A) a linear subspace of $\Re^2$

(B) convex

(C) convex and a linear subspace of $\Re^2$

(D) neither convex, nor a linear subspace of $\Re^2$

3. A school with n students has m clubs to which they can belong, and students are allowed to be members of multiple clubs. It is known that

(I) Each club has an odd number of members

(II) Every pair of clubs has an even number of common members (including 0)

Then it must be that :

(A) $m \geq n$

(B) $m \leq n$

(C) m and n can each be larger than the other, but $m \neq n$

(D) $m = n$

4. Let $n > 1$, and let S be the set of all $n \times n$ matrices whose entries are all chosen from the set {0, 1}. Then the sum of the determinants of all these matrices

(A) is equal to 0

(B) is positive but less than $n^{n/2}$

(C) can be positive or negative, but is bounded by n^n

(D) is either –1 or 1

5. $g : \Re \to [0, 1]$ is a non-decreasing and right-continuous step function such that $g(x) = 0$ for all $x \leq 0$ and $g(x) = 1$ for all $x \geq 1$. Let us define g^{-1} as follows, $g^{-1}(y) = \inf \{x \geq 0 \,|\, g(x) \geq y\}$. Which of the following is true about g^{-1} :

(A) is a continuous function

(B) is right-continuous but not left-continuous

(C) is left-continuous but not right continuous

(D) neither left-continuous, nor right continuous

The next Two questions (6 and 7) pertain to the following :

Four cities A, B, C, D are located as vertices of a square ABCD, and are connected by roads that form the four sides of the square. Mr. Rand Walker travels thus: if he is at city i ($i \in \{A, B, C, D\}$) in period t, then he randomly, with probability 1/2 each, moves to one of the two vertices/cities that are adjacent to city i in period $t + 1$.

6. If Mr. Walker is at city A at time $t = 0$, then the respective probabilities with which he is at the cities A, B, C, D in period $t = 10$ are :

(A) 1/4, 1/4, 1/4, 1/4

(B) 0, 1/2, 0, 1/2

(C) 1/2, 0, 0, 1/2

(D) 1/2, 0, 1/2, 0

5. $g : \Re \to [0, 1]$ is a non-decreasing and right-continuous step function such that $g(x) = 0$ for all $x \leq 0$ and $g(x) = 1$ for all $x \geq 1$. Let us define g^{-1} as follows, $g^{-1}(y) = \inf \{x \geq 0 \mid g(x) \geq y\}$. Which of the following is true about g^{-1} :

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6. If Mr. Walker is at city A at time $t = 0$, then the respective probabilities with which he is at the cities A, B, C, D in period $t = 10$ are :

(A) 1/4, 1/4, 1/4, 1/4

(B) 0, 1/2, 0, 1/2

(C) 1/2, 0, 0, 1/2

(D) 1/2, 0, 1/2, 0

7. If Mr. Walker is at city A at time $t = 0$, then what is the probability that he never visits city A again till (including) period $t = 10$?

(A) $(1/2)^5$ (B) $(1/2)^{10}$

(C) $(3/4)^5$ (D) $(3/4)^{10}$

8. Consider a railway signaling system : a signal is received by station A from the traffic control office and then transmit it to station B. Suppose that at the origin (traffic control office) signal can be yellow or red with probability 4/5 and 1/5 respectively. The probability of each station receiving the signal correctly from its predecessor is 3/4. If the signal received at station B is yellow, then the probability that the original signal was yellow is

(A) 22/23 (B) 11/20

(C) 20/23 (D) 9/20

9. Suppose X has a normal distribution with mean 0 and variance σ^2. Let Y be an independent random variable taking values –1 and 1 with equal probability. Define $Z = XY + X/Y$. Which of the following is true ?

(A) $\text{Var}(Z) > \sigma^2$

(B) $\text{Var}(Z) < \sigma^2$

(C) $\text{Var}(Z) = \sigma^2$

(D) $\text{Var}(Z)$ can be greater than or smaller than σ^2

10. According to the Ricardian equivalence proposition, a reduction in the current (lump sum) taxation on household income

(A) would reduce current consumption, but leave future consumption unaffcted

(B) would reduce future consumption, but leave current consumption unaffected

(C) would reduce both current and future consumption

(D) would leave both current and future consumption unaffected

11. Tobin's q theory suggests that firms will find it profitable to invest when the value of Tobin's q is :

(A) greater than zero (B) less than zero

(C) greater than unity (D) less than unity

The next Four questions (12 to 15) pertain to the following :

Consider an agrarian economy consisting of two single-membered households. The households are engaged in own cultivation using their family land, labour and capital. Each household is endowed with 1 acre of land and 1 unit of labour. However the two households differ in terms of their initial capital endowment (K_0^R and K_0^P), where R denotes the relatively richer household and P denotes the poorer household. Assume that $2 < K_0^R < 4$, and $0 < K_0^P < 1$. The households have access to two technologies, which are specified by the following production functions :

Technology A : $Y_t = (N_t L_t)^{1/2} (K_t)^2$;

Technology B : $Y_t = (N_t L_t)^{1/2} (K_t)^{1/2}$.

where N_t represents land (in acres), L_t represents labour, and K_t represents capital respectively.

The households choose the technology that gives them higher output (given their land, labour and capital stock) in any period t. In every period they consume half of their total income and save and invest the rest, which adds to the next period's capital stock. Land and labour stock remain constant over time. Existing capital stock depreciates fully upon production.

12. Given their initial factor endowments, the technology choices of the rich household and the poor household respectively are as follows :

(A) Household R chooses technology A; household P chooses B

(B) Household R chooses technology B; household P chooses A

(C) Both households choose technology A

(D) Both households choose technology B

13. In the short run, the average capital stock in the economy ($\bar{K}$) evolves according to the following dynamic path :

(A) $\frac{d\bar{K}}{dt} = \frac{1}{4}\left[\left(K_t^R\right)^{\frac{1}{2}} + \left(K_t^P\right)^2 - 2\left(K_t^R + K_t^P\right)\right]$

(B) $\frac{d\bar{K}}{dt} = \frac{1}{4}\left[\left(K_t^R\right)^2 + \left(K_t^P\right)^2 - 2\left(K_t^R + K_t^P\right)\right]$

(C) $\frac{d\bar{K}}{dt} = \frac{1}{4}\left[\left(K_t^R\right)^{\frac{1}{2}} + \left(K_t^P\right)^{\frac{1}{2}} - 2\left(K_t^R + K_t^P\right)\right]$

(D) $\frac{d\bar{K}}{dt} = \frac{1}{4}\left[\left(K_t^R\right)^2 + \left(K_t^P\right)^{\frac{1}{2}} - 2\left(K_t^R + K_t^P\right)\right]$

14. In the long run :

(A) income of both households grow perpetually

(B) income of household R grows perpetually while income of household P approaches a constant

(C) income of household P grows perpetually while income of household R approaches a constant

(D) income of household R grows perpetually while income of household P falls perpetually

15. If, at the end of the initial time period, the households were given a *choice* to spend their savings in buying more land instead of investing in capital stock :

(A) both households would have bought more land

(B) both households would have still invested in capital

(C) Household R would have still invested in capital but household P would have bought more land

(D) Household P would have still invested in capital but household R would have bought more land

The next Two questions (16 and 17) pertain to the following :

Consider an exchange economy with two agents, 1 and 2, and two goods, X and Y . There are 6 units of X and 4 units of Y available. Agent 1 has the utility function $u_1 = \min\{x_1, y_1\}$ and agent 2 has the utility function $u_2 = x_2 + y_2$. An allocation is denoted by (x_1, y_1), (x_2, y_2), where (x_1, y_1) is agent 1's consumption bundle and (x_2, y_2) is agent 2's consumption bundle, and $x_1 + x_2 = 6, y_1 + y_2 = 4$.

16. Which of the following allocations is *not* Pareto efficient ?

(A) (2, 0), (4, 4) (B) (4, 4), (2, 0)

(C) (2, 2), (4, 2) (D) (0, 0), (6, 4)

17. The set of equilibrium prices of this exchange economy is given by (prices of X and Y are denoted by px and py respectively) ?

(A) $\left\{px > 0, py > 0 \mid \frac{px}{py} \leq 1\right\}$

(B) $\left\{px > 0, py > 0 \mid \frac{px}{py} = 1\right\}$

(C) $\left\{px > 0, py > 0 \mid \frac{px}{py} \geq 1\right\}$

(D) Insufficient information to conclude

18 The market for widgets has the demand function $Q = 100 - 10P$, where Q is quantity demanded and P is the price in rupees. There are ten price taking firms in the market, each having a cost function $c(q) = \frac{1}{2}q^2$, where q is the firm's own output. There is no new entry. Suppose the government imposes an excise tax of Rs. 2 per unit of widgets, to be paid by sellers. The equilibrium market price is :

(A) 7
(B) 6
(C) 5
(D) 2

19. A consumer has utility function $u(x_1, x_2) = \min\{3x_1 + x_2, x_1 + 2x_2\}$. Prices of the two goods are p_1 and p_2 respectively. The consumer will buy positive quantities of both goods if and only if the price ratio p_1/p_2 is :

(A) greater than 3
(B) between $\frac{1}{2}$ and 3
(C) between 2 and 3
(D) less than $\frac{1}{2}$

20. Utility function of a consumer over three goods X, Y and Z as $U = y \min\{x, z\}$. Prices of the all three goods are the same in the market. Three discount deals are available, which are as follows ;

Deal I : Get 1 unit of Z free when you buy 1 unit of good X

Deal II : Get 1 unit of Z free when you buy 1 unit of good Y

Deal III : Get $\frac{1}{2}$ unit of X and $\frac{1}{2}$ unit of Z free when you buy 1 unit of good Y

Which of these deals should the consumer choose ?

(A) Deal I
(B) Deal II
(C) Deal III
(D) All three deals are equally good

21. Two widget producers, A and B, operate in perfectly competitive input and outp markets. Firm A uses capital (k) and labour (l) to produce widget; its productio function is $f_1(k_1, l_1) = (k_1 l_1)^{1/3}$. Firm B uses only labour; its production functio is $f_2(l_2) = \sqrt{l_2}/(1 + k_1)$. Efficiency of input allocation can be improved by :

(A) imposing a tax on capital use

(B) merging Firm A and B

(C) providing a subsidy on labour use to firm A

(D) all of the above

The next Two questions (22 and 23) pertain to the following :

A student has the opportunity to take a test at most thrice. The student knows that each time she takes the test, her score is an independent random draw from the uniform distribution on the interval [0, 100]. Each time the student takes the test and learns her score, she can either stop and accept it as her of offcial score, or she can discard the result and retake the test. However, after the third attempt, the student has no more opportunities to retake the test. In that case, her score on the last (i.e., third) try will be her official score. The student's objective is to maximize her expected offcial score.

22. If the student follows an optimal plan, her final expected score before taking any of the tests is approximately :

(A) 30 (B) 50

(C) 70 (D) 90

23. Now consider the case when the university allows a student to take the final test only when her score is below 40 in the second test. The student retains the choice to stop or retake after the first attempt. The student will decide to be retested after the first test if and only if her score is less than :

(A) 37.5 (B) 50

(C) 62 (D) 67.5

The next Two questions (24 and 25) pertain to the following :

Consider a homogeneous goods market with two firms. Let x_1 and x_2 be the quantity choices of Firms 1 and 2 respectively. The total quantity is $X = x_1 + x_2$. The inverse demand function is $P = \alpha - \beta X$, where P is the market price and α and β are the intercept and slope parameters respectively. Both firms have the same marginal costs denoted by c. Suppose $\beta > 0$, and $0 < 3c < \alpha$.

24. Suppose firm 1 has an objective of maximizing revenue and firm 2 has an objective of maximizing profit. Both firms choose quantities simultaneously to maximize their respective objective functions. Then :

(A) Firm 1 has larger market share and larger profit

(B) Firm 2 has smaller market share but larger profit

(C) Firm 1 has smaller market share but larger profit

(D) Firm 2 has larger market share and larger profit

25. Suppose both firms become revenue maximizer. Then :

(A) both will produce more than Cournot output

(B) both will produce less than Cournot output

(C) both will produce the perfectly competitive output

(D) both will produce more than the perfectly competitive output

26. If the following equation is estimated using OLS, and a 95% confidence interval for β_1 is constructed, then which of the following is true ?

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \epsilon$$

(A) The OLS estimate of β_1 lies in the given interval with 95% probability

(B) The true value of β_1 lies in the given interval with 95% probability

(C) In repeated sampling 95% of the times the confidence interval will contain the true value of β_1

(D) Both (B) and (C)

27. What term would best describe the shape of the given boxplot ?

(A) Right skewed

(B) Left skewed

(C) Uniform

(D) Normal

28. The Vitamin C content of a particular brand of vitamin supplement pills is normally distributed with mean 490 mg and standard deviation 12 mg. What is the probability (approximately) that a randomly selected pill contains at least 500 mg of Vitamin C ?

(A) 0.8

(B) 0.2

(C) 0.025

(D) 0.55

29. A sample of 100 cows is drawn to estimate the mean weight of a large herd of cattle. If the standard deviation of the sample is 100 kg, what is the approximate maximum error in a 95% confidence interval estimate ?

(A) 10

(B) 20

(C) 30

(D) 40

30. Recent studies suggest that the migration to Indian cities from rural regions (MIG) can be explained by quality of life (QL), state income as a ratio of aggregate Indian income (Y) and the ratio of state employment to overall employment in India (E). Using data for the 29 Indian states, the following model is estimated :

$$\widehat{MIG} = -4.2 + 1.2QL - 0.6Y - 0.8E$$
$$(0.9) \quad (0.8) \quad (0.05) \quad (0.02)$$

The figures in parentheses are standard errors. The t-statistic for the null hypothesis that the quality of life index does not impact migration is :

(A) 0.8

(B) 0.9

(C) 1.5

(D) Insufficient information to calculate

31. Consider the following estimated regression relating expenditures on food (Y_i) to income (X_i)

$$\hat{Y}_i = 145 + 0.3X_i - 0.1(X_i - X_i^*)D_i$$

where i denotes the individual; $X_i^* = 500$, this threshold distinguishes low-income from high-income individuals, and D_i takes value 1 if $X_i > X_i^*$ and is 0 otherwise. All the estimated coefficients are significant at the 5 percent level. Which of the following statements is *false* ?

(A) The marginal propensity to consume for people with low income is 0.3, and is lower for those with higher incomes; this makes sense as it is in accordance with Engel's Law

(B) This is a differential-slope, common-intercept dummy variable formulation with an additional restriction that leads to a kinked Engel curve

(C) This is a standard, differential-slope, common-intercept dummy variable formulation

(D) Using this formulation yields predicted expenditure on food for people at income level of 490 that is not very different from those for people at income level of 510 (estimates lie within 5 percent of each other)

32. The parameters of the following multiple regression model have been estimated using ordinary least squares.

$$Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$$

Denote the estimated residuals from this regression as e. Which of the following statements is *false* ?

(A) The R-squared from the regression of e on a constant, X_2 and X_3 is zero

(B) The R-squared from the regression of Y on a constant and $\hat{Y}$ is the same as in the original regression ($\hat{Y}$ stands for the predicted value of Y)

(C) The slope coefficient from the regression of Y on a constant and $\hat{Y}$ is 1 and the intercept is zero

(D) The R-squared from the regression of Y on a constant and e is the same as in the original regression

33. In a multiple regression model with four explanatory variables, with standard assumptions, estimated using ordinary least squares, all the coefficients turn out to be insignificant although the overall R-squared is high and the associated F-statistic is significant. Also, the pair-wise correlations amongst the four explanatory variables are all low, and range between 0.1 and 0.2, but are not statistically different from zero. Which of the following statements is *false* ?

(A) Even though the pair-wise correlations between the explanatory variables are low, since they are (individually) statistically different from zero, OLS coeficients are likely biased

(B) This is likely a case of multicollinearity even though the pairwise correlations are low

(C) If the analyst is only interested in making forecasts then insignificance of coeficients is per se not a problem since the F-statistic is significant and R-squared is high

(D) Dropping a variable may improve significance of the remaining coefficients but they may be biased

34. Instead of estimating a true cost function, which is described as a quadratic, where costs are regressed on an intercept, output and a quadratic term in output, a researcher estimates a linear function by regressing costs on an intercept and output. The estimates from the linear cost function are likely to :

(A) have autocorrelated residuals

(B) be a biased estimate of marginal cost, even though there is no exact linear relationship between the linear and quadratic terms in output

(C) both (A) and (B)

(D) neither (A) nor (B)

35. Using data on class size (CS) and average test percentage (TP) from 101 classes, the following OLS regression is estimated :

$$\widehat{TP} = 96.4 - 1.12\ CS$$

R-squared is 0.1 and SER (Standard Error of the Regression) is 5. What is the sample variance of test percentages across the 101 classes ?

(A) 27.5 (B) 22.5

(C) 5.0 (D) 2.5

36. In the OLS regression $Y_i = \beta_1 + \beta_2 X_i + u_i$, suppose the coeficient of determination is estimated to be 0.6. We now transform the variables such that $Y_i^* = 0.5Y_i$ and $X_i^* = 0.75X_i$, and re-run the regression. The coefficient of determination is now :

(A) 0.6 (B) 0.4

(C) 0.9 (D) 0.3

37. The OLS regression of infant birth weight (BWT) on mother's age in years (AGE) and years of mother's education (EDU) is :

$$\widehat{BWT} = 2600 + 2.3AGE + 26EDU$$
$$(97) \quad (3.5) \quad (8)$$

where standard errors are reported below the estimated coefficients. Sample size is 1000 and R-squared is .015. Sample information is provided in the table below :

	BWT	AGE	EDU
Mean	3000	25	10
Standard deviation	500	5	2

A one standard deviation change in AGE is associated with an x standard deviation change in birth weight, where x is :

(A) 0.046 (B) 0.007

(C) 0.023 (D) 0.035

38. If $f : \Re \to \Re$ is a continuous function ($\Re$ is the set of real numbers) and E is an open subset of $\Re$, then the set $\{x \in \Re \mid f(x) \in E\}$ is necessarily

(A) open (B) closed

(C) neither open, nor closed (D) open and closed

39. If X is an $n \times n$ non-singular matrix such that $XX^T = X^TX$ (X^T denotes the transpose of X). Let $Y = X^{-1}X^T$ where X^{-1} is the inverse of X. Then YY^T is equal to

(A) I + Y (I is the identity matrix) (B) I

(C) Y^{-1} (D) Y^T

40. Suppose that $g(x)$ is a twice differentiable function and $g(1) = 1$; $g(2) = 4$; $g(3) = 9$. Which of the following is necessarily true ? First derivative and second derivative of g (with respect to x) are represented by g' and g'' respectively.

(A) $g''(x) = 3$ for some $x \in [1, 2]$

(B) $g''(x) = 5$ for some $x \in [2, 3]$

(C) $g''(x) = 2$ for some $x \in [1, 3]$

(D) $g''(x) = 2$ for some $x \in [1.5, 2.5]$